

GRAPHICAL INTEGRATION OF ROBOT PROGRAMMING  
AND SEQUENCE PLANNING FOR  
MECHANICAL ASSEMBLY

CENTRE FOR NEWFOUNDLAND STUDIES

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YUNQING GU









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**Graphical Integration of  
Robot Programming and Sequence Planning  
for Mechanical Assembly**

by  
Yunqing Gu

A thesis submitted to the School of Graduate Studies  
in partial fulfillment of the requirements  
for the degree of  
MASTER OF SCIENCE

Department of Computer Science  
Memorial University of Newfoundland  
Newfoundland, Canada  
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by

Yunqing Gu

## Dedication

*To my parents  
for their encouragement and support  
throughout the course of my education.*

# Abstract

A major problem plaguing programmable robot-based assembly systems today is the lack of communication between robot programming and assembly planning. Whereas a robot learns to perform assembly tasks through *programming*, it assembles mechanical parts into a product by following the assembly sequence determined by *planning*. Robot programming and assembly planning have been dealt with mostly as two separate research topics. Nevertheless, the internal representation of parts and their topological and geometrical relationship required by planning could be automatically synthesized from the information obtained in programming. This observation becomes a view in advanced assembly automation systems that integrate robot programming and assembly planning for better performance and higher efficiency.

This thesis addresses the issues of graphical integration of the robot programming process and the sequence planning process for mechanical assembly. It treats the robotic assembly sequencing as a motion-planning problem with special constraints. During the programming phase, the order of effective assembly actions is built once a robot effectively performs an assembly task. As a result, the information of an assembly scene and ordered robotic actions can be produced and stored. In the unified system presented in the thesis, the sequence planning process directly retrieves this information for automatic and fast planning. Since analyzing ordered robotic actions

and executing partial geometric checks are usually much fast than performing full geometric checks, this integrated approach offers significant computational advantages in comparison with other ‘full-automatic’ planning approaches. In addition, the approach feeds all the feasible sequences generated from the planning process to the programming process, and provides the ‘automatic re-programming’ feature to the programming process. As a link between the programming and planning processes, this integrated approach presents a richer form of communication between them, which is necessary to efficiently solve real-world robot-based automation problem. The validity of this approach is justified with a prototype system, namely INTEG. Experimental results are also given in the thesis that include examples varying from simple to complex assembly tasks.

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# Chapter 1

## Introduction

Tomorrow's product systems need a radical change to the conduct of manufacturing business. With the revolutionary advances in information, telecommunication, and computing technologies, the impact on the Computer-Integrated Manufacturing(CIM) is shifting the tradition 'factory integration' philosophy to a 'virtual factory' management philosophy [31, 25, 56, 39]. A variety of supporting techniques for achieving the paradigm of 'virtual factory' is promising and has been attracted much attention. *Concurrent Engineering*(CE) is defined as a systematic approach to the integrated, simultaneous design of products and processes, including manufacturing and support [48, 29]. *Virtual Manufacturing*(VM), in comparison, is defined to be an integrated, synthetic manufacturing environment exercised to enhance all levels of decision and control [13, 14]. The work presented in this thesis, towards the application of CE and VM techniques to the development of robot-based assembly systems, investigates the integration of robot programming and sequence planning for assembly automation. The following sections of this chapter describe thesis scope and motivation, research issues and objectives, and the overall structure of the thesis.

## 1.1 Thesis Scope

This thesis addresses the problems of exploring robot-based assembly with fundamental capabilities required by an intelligent and autonomous system. Although robot-based assembly automation covers a wide range of processes [45, 8, 40, 18], the thesis concentrates mainly on the integration of the robot programming and sequence planning processes for 'optimal re-programming'. This is because they are two basic processes that not only robot-based assembly automation requires but also the concept of *concurrent engineering* applies to. While *robot programming* is a process of improving robotic operations [46], *assembly planning* is a process of determining a set of instructions for mechanically assembling a product from a set of subcomponents [35]. These two research areas have emphases of their own, and have been dealt with as two different topics. This thesis posits at the interlacing problems – 'programming for planning' or 'planning for programming'. It endeavors to seek the intrinsic connections between them, and to incorporate them so that design, programming, planning, scheduling and re-programming activities can be better practiced.

## 1.2 Motivation

The research of an integration of robot programming and assembly sequencing is initiated for at least three reasons. First, recent strides toward *Concurrent Engineering* call for a tighter integration of assembly planning with robot task planning [58, 24]. This is because a significant amount of assembly cost can be cut by re-planning the assembly tasks, and re-programming robots. Secondly, there are close relationships



between robot programming and sequence planning. In order to choose an assembly plan, as an example, one must know how the tasks will be executed; but the way tasks are executed depends on the choice of the assembly sequence. As a result, the output of assembly planners should be compatible with what is required by task and motion planners [36]. Finally, programming and planning can be performed more efficiently and can be further automated if they are compatible with each other. For instance, there are many common cases in which the re-design of a robot program is necessary or desirable. In these cases, '*automatic re-programming*' is preferable. The premise of '*automatic re-programming*' is that a robot can practice an assembly plan off-line, use that experience to generate other feasible assembly plans, and re-program itself for effectiveness and productivity [72].

The exchange of data between object modeling, assembly planning, robot programming, and motion planning is an open question although considerable efforts have been put in each of these areas. Publications on integrated models for robot programming and sequence planning are still rare, and the majority of related work rests either in assembly sequence planning [5, 47, 35, 50, 45], where the manipulator constraints or the virtual process of generating assembly sequences are not concerned, or in off-line robot programming [34, 23, 26, 12, 62], where the functions of assembly sequence planning are ignored. Discussions on '*automatic re-programming*' as a complex problem-solving activity are uncommon, which suggests that generic robot programming techniques usually do not provide '*automatic re-programming*' features. After an initial period of developing stand-alone programming and planning systems, the need of integrating them became apparent. Advanced industrial manufacturing

requires the extension of existing robot programming and planning techniques, and further exploration of new techniques. The result will be the development of new robot programming techniques.

### 1.3 Research Issues and Objectives

Two main objectives underlie this research: one is to investigate an integrated approach that makes the robot programming and planning processes as user-friendly as possible; the other is to design an integrated model for off-line robot assembly and to use it in robot programming and sequence planning. As the integration of programming and planning processes is focused in the thesis on the extension of off-line programming systems by adding functions of sequence planning, the following three questions are fundamental:

1. What information is required to develop a schema for the integration of the robot programming process and the assembly planning process? Since generally a very large amount of information is required to synthesize these two processes, it is necessary to identify and extract the most relevant information for their integration.
2. What kind of methodology can be used to create a robot workcell that is capable of programming a robot off-line? The method must be convenient for programming a robot, and effective for solving assembly sequencing problems later.

3. How to design and develop an assembly planner that facilitates sequence planning in an off-line programming system? Assembly planning is strongly dependent on manipulation machines. For example, an assembly plan that is ideal for assembling may not be suitable for machining. Hence, approaches to computer-aided assembly planning are influenced by the type of selected robot task processes.

These three questions are also the problems that this thesis research is to address. The first problem needs an integrated model for robot programming and sequence planning. Such a model is intended to integrate diversified expertise and to improve the flow of information from the programming and sequencing processes. The second problem needs an understanding of relations between robot and human so as to develop a scheme to program a robot off-line. The scheme should facilitate both interactive programming and automatic sequencing. And the third problem needs to study the effect of robot assembly operations on assembly planning at a higher level of abstraction without certain elements of detail. This research explores the possibility of implementing assembly sequencing algorithms for problems of a manageable size with special robot operations.

To sum up, this thesis research is to establish a mapping between robot programming and sequence planning and to identify basic issues related the systematic synthesis of these two processes. Relevant issues are taken as the specific objectives of the thesis study. In order to achieve the objectives, an integration of both robot programming and sequence planning is essential. The ultimate objective of this research is to develop an integrated approach of creating intelligent and autonomous robot-based assembly systems with a high degree of flexibility, which can be used

directly to improve assembly processes of manufacturing.

## 1.4 Thesis Structure

There are seven chapters in this thesis. The structure of the thesis reflects the structure of the research itself. A research introduction has been given in **this chapter**, while the rest of this thesis is organized as follows.

**Chapter 2** presents a literature review of related areas. It has two sections. The first section describes various approaches to robot programming, especially to off-line programming. And the other section describes existing approaches to computer-aided assembly planning, with discussion about their limitations.

In **Chapter 3**, the thesis proposes an integrated approach to robot-based assembly automation through an integration model. This approach features an 'automatic re-programming' functionality. Problem-domain concepts, including robotic assembly definition and representation, are also given in this chapter.

**Chapter 4** first gives a formal geometric definition to the robot assembly problem, then discusses the direct kinematics problem which determines the position and orientation of a robotic simulator from a given state of all the joints, and finally reaches to a definition of the 'teach-in' process.

**Chapter 5** describes an approach to the extension of off-line programming systems by adding an assembly sequencing feature. As the core of this thesis, this chapter identifies the basic meaning of robotic assembly sequencing. And then it presents an assembly planner, which works in a 'semi-automatic' fashion and generates all the

feasible assembly sequences for a robotic assembly from the result of off-line robot programming.

**Chapter 6** describes the development of a proof-of-concept prototype, namely INTEG, for robotic assembly programming and sequence planning. It also provides a set of experiment examples to demonstrate the operation of INTEG.

The last chapter, **Chapter 7**, summarizes the main research accomplishments of this thesis, describes the anticipated impact, and highlights several contributions to computer-aided manufacturing and robot-based assembly automation. This chapter concludes with a discussion of the current limits of the thesis research and the directions for future research.

## **Chapter 2**

### **Review of Related Research Work**

This chapter presents a summary of research work in the area related to the topic of this thesis. In the last two decades, many researchers have addressed robot programming and assembly planning in different manufacturing applications. An overview of their work and the relative techniques is particularly useful to developing a schema for the integration of robot programming and sequence planning. The review starts with robot programming and then assembly planning. A number of approaches adopted or developed for off-line programming and assembly planning are also presented with discussions about their limitations.

#### **2.1 Robot Programming**

A robot is a general purpose programmable manipulator. It can be used to perform tasks that it has been ‘taught’ to do. The process of teaching a robot is also called ‘programming’ the robot [38, 46, 44, 41]. There are different programming methods available to meet the requirements of different programmer skills and the complexity

of the task, as well as the related productivity and safety issues. Teaching a robot is in fact to program it to perform a specific task. A large part of robot programming involves defining a path for the robot to follow. There are two main approaches of robot teaching: *on-line programming* and *off-line programming*. When using on-line methods, the robot itself has to be used during programming. In comparison, the off-line approach allows the user to program tasks on a different computer system and then download task application programs into the robot's control system. More discussions on on-line programming methods can be found in [71, 63]. The off-line methods, as of interest to this thesis research, are discussed below.

### 2.1.1 Off-line Programming

*Off-line programming* suggests developing robotics control programs away from the robot system and perhaps on a different computer [63]. The idea has been around for years but it gained importance only recently due to the integration of simulation packages, computer-aided design, and computer graphics into one programming environment. There are more advantages when using off-line programming environment. Several reasons for programming robot off-line are listed as follows.

- **Reduction in robot down-time:** Using off-line programming, if an engineering change is made, the robot's programs may be developed off-line and loaded into the robot's system. Off-line programming makes programming possible even before the robot's work-cell is completely installed.
- **Ease of programming:** Off-line programming considerably reduces the time

required to create a program and makes programming changes much easier. In addition, the programmed task may be visualized on the computer screen before being used to operate the real robot.

- **Use of CAD systems:** The availability of CAD systems means that computers can acquire design data specifications and derive procedures to implement a working program for the robot. The cost effectiveness of using personal computers with CAD systems is a decisive factor in the popularity of off-line programming.
- **Visualization:** With the aid of computer graphics and CAD representation of robots, machine tools, and different objects, it is possible to generate an animated simulation of robotic tasks. This simulation of positional data makes it easy to generate accurate robot programs.

### 2.1.2 Approaches of Off-line Programming

There is a variety of different approaches to programming robots off-line. Depending on the application of a robot, different teaching methods may be used. These approaches are discussed below:

**Textual programming** *Textual programming* is often used in academic environments. A programmer stores a robot command sequence in a computer as a textual program [60]. It requires a long development period and expert programmers. A problem with this textual programming method is the difficulty to specify the movement points without the robot.



**Teach By Showing or PbD** The *‘teaching-by-showing’* method consists of making a robot, for example, by means of a remote control. The method requires human to show robot movements and then a robot would have to repeat these movements. It suits quite well to early simple robot applications. Using this method, Kuniyoshi *et al.*[70], Suehiro *et al.*[65], and Friedrich *et al.* [15, 16] created auto-programming methods which have been called *Programming by Demonstration* (PbD) or *Demonstration for Programming* (DfP). In PbD method, a human instructor demonstrates a task to a robot by performing it himself. However, PbD methods are based on vision and require a lot of computer power for the recognition of assembly states or the operator’s movements.

**Teleoperation** To explore hazardous environments, *teleoperation* methods have been proposed [43, 69, 1]. This method uses a master manipulator for teaching and a slave manipulator for execution. An engineer controls the master manipulator in a safe environment while monitoring the hazardous environment through a remote TV camera. By using this method, users can only teach a robot trajectory information. However, it is difficult to build a flexible robot system of planning and recovery capabilities.

**Automatic programming** *Automatic programming* tries to develop geometric reasoning systems which can generate textual programs to control a robot from geometric information given by geometric models and task specifications. It is a promising direction. However, there are many issues to be addressed before a complete automatic programming system becomes possible; Problems with this

method arise due to the high complexity of the tasks which require planning systems that are currently not available for industrial applications[15].

**VR approach** Takahashi *et al.*[67, 66, 17, 2] proposed a robot teaching interface which uses *Virtual Reality*(VR). An operator wearing a data glove device teaches an assembly task in a virtual workspace. A robot with sensors and macro command interpretation facility performs the assembly task in an actual workplace. This method requires not only a lot of hardware resources but also new methods to handle and to calibrate virtual objects.

**Interactive computer graphics** Recent advances in computer graphics technology have made it possible to simulate the teaching of robots [59, 26, 62]. Software systems with off-line programming capabilities are commercially available [27, 7, 34]. These systems have many advanced features: collision detection, signature models, and dynamic simulation, as well as the ability to write device control codes. But these systems often cost more than the hardware on which they are running, and also too much focus on graphical simulation since they lack the ability to integrate with the assembly planning process.

## 2.2 Assembly planning

*Assembly planning* primarily concerns the feasible sequences that assemble a product. The field of computer-aided mechanical assembly planning has emerged in recent years from the progress in artificial intelligence and robotics, especially in the areas of planning, and geometric and physical reasoning. The importance of automating

the planning of mechanical assembly is twofold. First, it gives industrial designers a tool with which they can assess their designs for easier assembly. It hence increases the efficiency of production process and drastically reduces redesigns due to manufacturing constraints. Second, it closes the gap between computer-aided design (CAD) and computer-aided manufacturing (CAM) by providing tools for automating the programming of robots and assembly work-cells. It therefore expedites the execution of these chores by reducing their cost and improving their quality[35]. Areas of assembly planning that are particularly relevant to this thesis research are as follows.

### 2.2.1 Assembly Representation

An assembly planning relies on the representation of assembly parts and assembly operations. Several methodologies for representing assembly plans have been proposed and utilized. They include representations based on directed graphs, and AND/OR graphs, on establishment conditions, and on precedence relationships [37]. Among them, the last one includes two types: precedence relationship between the establishment of one part connection and the establishment of another, and precedence relationship between the establishment of one connection and the states of an assembly process. The methods based on directed and/or AND/OR graphs are explicit representations since there is a mapping from the assembly tasks into the elements of the representations. And the others based on establishment conditions and precedence relationships are implicit representations because they consist of conditions that must be satisfied by the assembly sequences. A clear understanding of these different representations and of how one maps into the others is important in the

development of an assembly planner.

### 2.2.2 Sequence Planning

*Sequence planning* considers the order in which the component parts will be assembled [55]. There has been considerable research in assembly sequencing during the past decade. Early assembly sequences were mainly interactive sequence editors; geometric reasoning was then added to generate assembly sequences automatically[9, 21]. This research first resulted in generate-and-test sequences, with a module guessing candidate sequences and several geometric reasoning modules checking their feasibility. More efficient techniques were later proposed to replace the time-consuming generate-and-test[47]. Assembly sequencing has been shown to be intractable[21]. This negative result has led researchers to consider restricted, but still interesting types of assembly sequences. Typically most research so far has focused on computing either monotone sequences, where each operation generates a final subassembly, or two-handed sequences, where every operation merges exactly two sub-assemblies. Though restrictions vary slightly among the assembly sequences proposed so far, one is made in all of them: parts are uniquely defined by their nominal geometry.

### 2.2.3 Approaches of Assembly Planning

Assembly planning is also a topic that has been addressed in the literature. Two basic approaches of dealing with geometry have been prevalent [72]:

**Editor Planning** ‘*Editor*’ approach is to avoid the problem of assembly planning by

not requiring automated geometric reasoning. In the systems first developed by Boujault [22] and later improved by DeFazio and Whitney[10], geometric reasoning was done by a human interacting with a computer-based system. The user specifies assembly plans by answering a series of questions about the 'liaison' relationships between components.

**Automatic Planning** 'Full-automatic' approach is emerging as an approach that facilitates the automation of planning. It performs full geometric reasoning with traditional CAD representations. Much work can be found in [5, 47, 50, 28, 57, 21].

Both approaches are intended to work with realistic assemblies, and hence require realistic representations of the components. In the human-based ('editor planning') systems, this representation is a mental model of the assembly, most similar to a relation graph. However, manual analysis of the 'liaison' relationships between components is laborious and time-consuming. In the automated systems, on the other hand, this representation is a CAD model. Due to the complexity of automated geometric reasoning, measures have to be taken to either eliminate automatic reasoning or perform it as infrequently as possible. In practice, the computational automated analysis of assembly operations is time consuming.

## **Chapter 3**

# **Integration of Robot Programming and Sequence Planning**

This chapter proposes a new scheme of a robot-based assembly system that integrates the planning process with the programming process in a unified system with a common database. The proposed approach is initiated from a novel idea – providing off-line programming system with ‘automatic re-programming’ features [72]. In this approach, assembly representation has been considered as a manipulation planning problem [50]. The succeeding sections give a definition of the problem and present in detail the framework of this integrated approach.

### **3.1 Towards Integration**

The use of programmable robot systems has enabled a partial or complete automation of product assembly. The performance of off-line robotic assembly tasks requires an integration of an assembly information with an internal representation of parts, together with geometries and relationships. More systematic approaches to the au-

automatic synthesis of these information are needed to enhance their performance and enable their cost-effective implementation. This thesis investigates the integration of robot programming and sequence planning with an emphasis on exploring 'automatic re-programming'.

### 3.1.1 'Automatic Re-programming'

There are many common cases in which a re-design of robot programs is necessary or desirable. They have a fixed set of assembly tasks with only their order needing to be changed to adapt to the changing conditions in an environment. For example, a supplied assembly task sequence is usually not the optimal solution as users seldom consider all assembly task sequences before choosing a sequence. These tasks, therefore, may need to be re-programmed for product improvement. In addition, programming chores in manufacturing are time consuming and error-prone. For small batches of a production, the cost of programming weighs heavily in the total production cost. Moreover, the time spent in programming, when carried out manually, may excessively delay the actual production. The automation of those chore will expedite their execution, reduce their cost, and improve their quality. As a result, an advanced programming system needs to perform robot simulation concurrently with other planning processes and provide a feedback mechanism for 'automatic re-programming' whenever necessary.

The exploration of an 'automatic re-programming' feature requires an integrated model that allows a computer to generate all the feasible assembly sequences for a robotic assembly. These sequences are strictly from the simulation of robot's per-

forming the same task. A theoretical work on geometric assembly planning suggests that an assembly sequencing problem can be seen as a motion planning problem with multiple moving objects [50]. Assembly information can thus be acquired by either a motion planner or an assembly planner; and the most reliable way is to simultaneously take both and to appropriately integrate the assembly information from the two methods.

### 3.1.2 Relation to Other Work

An idea similar to the integrated approach of robot-based assembly automation has been presented and constructed through the aid of an assembly planner in robot-based CAM/CIM systems [45, 68]. Despite the incorporation of a feasible assembly planner based on the pre-planning conditions associated with planning before programming, the system was not capable of ‘automatic re-programming’. The functionality of ‘automatic re-programming’ requires an assembly planner that uses the pre- and post-conditions associated with robotic assembly tasks to construct sequential high-level assembly analysis. Earlier work [35, 50] on assembly planning systems facilitated the subsequent manufacture. However, they do not suit to the integration with off-line robot programming systems, in which assembly planners have to take into account the capabilities and limitations of task and motion planners.



## 3.2 An Integrated Model

The basic structure of the proposed integrated model (shown in Figure 3.1) consists of a programming model and a planning model. The programming model offers the user a graphics interface to program a robot off-line, and the planning model generates all the feasible robotic assembly sequences for automatic re-programming. As a means of reaching to integration, information regarding to an assembly scene and an order of robotic actions is directly stored in the programming phase, and then retrieved and re-used in the planning phase to help fast sequence generation. For instance, a precedence expression of assembly operations can be used to find other valid operation orders. In such a way, the check of geometric feasibility can be significantly reduced by introducing simple robot action analysis in the planning phase. Furthermore, all the feasible sequences generated from the planning model can be fed back to the programming model for 'optimal' programming.

Many approaches are possible for the development of a programming model and a planning model. They include 'teaching-by-showing', 'VR', and 'automatic programming' approaches for the programming, and 'editor' and 'full-automated' approaches for the planning. In this thesis, the programming model and planning model are developed with an 'interactive graphics' programming approach and a 'semi-automated' planning approach respectively. While detailed discussions on these two models will be provided in the succeeding chapters, a brief introduction is presented as follows.

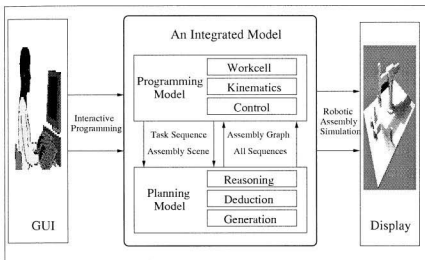


Figure 3.1: The basic structure of an integrated model.

### 3.2.1 Programming Model

The programming model is basically composed of a workcell module for the geometric definition of robot and component parts, a kinematics module for controlling manipulator positioning, and a control module for specifying robotic assembly tasks. This programming model creates a simple, useful, and problem-oriented programming interface.

### 3.2.2 Planning Model

The assembly planner model, concerning both assembly and manipulator constraints, composes of three basic components. They are the reasoning, deduction, and generation modules. The reasoning module performs geometric reasoning and detects geometric constraints between two parts; the deduction module deducts assembly

precedence constraints among parts; and the generation module generates a feasible assembly tree based on a set of assembly precedence relationships.

### 3.3 Robotic Assembly Problem and Representation

#### 3.3.1 Robotic Assembly Definition

A *mechanical assembly*, or assembly for short, is a composition of interconnected parts forming a stable unit. Each part is a solid object, and parts are interconnected whenever they have one or more surfaces in contact. In the thesis, a *robotic assembly* is referred to an assembly in which the product is assembled by a robot arm assigned with certain assembly tasks. In most robot-based assembly systems, part positioning is carried out sequentially, with only one part or a subassembly positioned at a time. A stable workstation, denoted by *tab* or  $P_0$ , is a place at which a robotic assembly takes place.  $P_0$  or *tab* is always assumed to exist in a robotic assembly system. Figure 3.2 shows a robotic workcell for robotic assembly.

A *robotic assembly process* is therefore assumed to consist of a succession of robotic assembly tasks, each of which in turn consists of joining sub-assemblies to form a larger subassembly. It is also assumed that exactly one part is 'pick-and-placed' on other sub-assemblies or a stable workstation (i.e. table) at each assembly task, and that after parts have been put together, they remain together until the end of the assembly process. More formally, a robotic assembly is defined as follows:

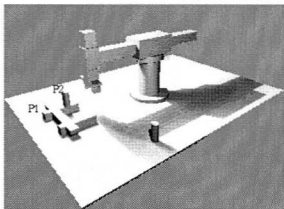


Figure 3.2: A robotic workcell for robotic assembly.

**Definition 3.1 (Robotic Assembly)** *A robotic assembly is an assembly  $\mathcal{A}(P, \mathcal{T})$ , in which the product is assembled from a set of parts  $P$  by a robot arm assigned with an ordered assembly tasks  $\mathcal{T}$  in a 3-dimensional workspace  $\mathbb{W}_A$  and no obstacles exist. In the assembly,  $\mathcal{P} = P_0 + P$ , where  $P_0$  denotes a stable workstation and  $P = \{P_1, P_2, \dots, P_n\}$  is the set of parts of assembly, and  $\mathcal{T} = \tau_1 \tau_2 \dots \tau_n$  is an ordered ‘pick-and-place’ operation task for assembly.*

The above definition of a robotic assembly works with a general-purpose *linear assembly*. Other robot operations such as one-hand ‘side-insert’ and ‘multi-hand’ operations, which are allowed in a wide application domain, are not covered by Definition 3.1.

### 3.3.2 Assembly Programming

In robot-based assembly systems, the way that a manipulator handles component parts determines the efficiency of geometric operations. It is necessary to consider about collision-free trajectory between two objects which are the base part with

its fixture and the secondary part with its handling device. With the assumption that one of the robotic assembly plans is known in advance, the user programs a robot for robotic assembly off-line by following the given assembly sequence. *Robot programming* then specifies the course of robotic actions and provides collision-free trajectory to achieve assembly goal. Currently, robot actions are restricted to 'pick-and-place' operations. In such a case, the motion of a part is simply a translation in the x-y plane of a coordinate system. These assumptions commonly exist in some applications such as printed wiring assembly or loading assembly. They also reflect the limited motion freedom of industry robots.

### 3.3.3 Assembly Sequencing

A *robotic assembly plan* consists of a set of assembly tasks with ordering constraints among its elements. The assembly planning problem is actually a manipulation planning problem, in which the initial configuration satisfies Definition 3.1 and the result is an assembled product. *Assembly sequencing* focuses on the order that 'pick-and-place' operations will follow to define a feasible assembly. For instance, it decides in what order the component parts should be needed for an assembly. The generated assembly sequences are also under various constraints regarding the robot and its surrounding world related to the underlying goal.

### 3.3.4 Robotic Assembly Representation

Robot programming and sequence planning requires a representation that is independent of the robot that will be performing robotic assemblies. This representation

should also be used later for assembly sequence planning. To demonstrate the concept without being hindered by other computational problems, the manipulated objects will be limited to polyhedral objects. A moving object is attached to other stationary environmental objects to form a particular assembly relation with each other. Assembly relations are in turn defined with face contacts between a manipulated object and its stationary environmental objects. The essential goal of an assembly task is to establish a new face contact between a manipulated object and its environmental objects. For example in Figure 3.2, the goal of placing a cylindrical object  $P2$  onto a stationary subassembly  $P1$  is to make a face contact so that the bottom face of the  $P2$  attaches to the top face of  $P1$ . The type of assembly relations becomes the central representation for defining assembly task models. This thesis research employs face contact relations [33] as its basic representations. It describes an assembly task as a transition from pre-assembly relations to post-assembly relations.

### 3.3.5 Component Representation

One way to represent objects is to use geometric data supplied by CAD models of objects. The complexity of this task depends on the complexity of the models and their representations. The simplest case is with polyhedral objects [4]. A more complex class of models involves objects produced by combining "nameable" surfaces such as planes and cylindrical surfaces, and solid shape primitives such as boxes, cylinders, and spheres [51]. The most complex case addressed to date involves objects with parameterized surface patches expressed as bicubic equations [52]. Such models are called boundary representation (B-rep) models. Most of mechanical parts can be

represented with the CSG formalism [20, 51] or feature-based merging [6, 73].

### 3.3.6 Assembly Constraint

The integrated model considers the generated assembly plans are *sequential* and *monotone*, without *coherence* constraints. The assembly planner adds in another constraint on linearity. A plan is *linear* if no more than one part is moved at a time [28]. Such a plan contains no sub-assemblies and is constructed entirely in a single fixture. There are many common assemblies in which the use of sub-assemblies is desirable. For such assemblies, the parts can be first broken down into sub-assemblies, and each subassembly is then treated as a separate linear plan.

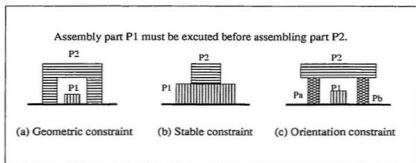


Figure 3.3: Illustrations of assembly constraints for robotic assembly.

A robotic assembly process has to follow some logic in order to avoid useless re-orientations or repeated tool changing. Assembly planning is also subjected to *assembly constraints*. The order of assembly operations depends on the feasibility of assembly steps. The following list specifies some of the most common constraints applied to robotic assembly tasks:

- **Geometric constraint:** When mating two parts there must be at least one collision-free trajectory allowing to bring the two parts in contact. For example, Figure 3.3 (a) shows two parts  $P1$  and  $P2$  in their goal position. Since  $P2$  is 'external of'  $P1$ , it is necessary to assemble part  $P1$  first. A notation  $PC(P1, P2)$  is used to indicate such a precedence relation. It means that part  $P1$  must be assembled before part  $P2$ .
- **Stable constraint:** Each part produced in the course of an assembly process must be stable, and all the involved components have a fixed spatial relationship between each other. As shown in Figure 3.3 (b), to place part  $P2$  on top of part  $P1$ , part  $P1$  must be assembled first. Again, the resulting precedence constraint between  $P1$  and  $P2$  is denoted by  $PC(P1, P2)$ ;
- **Orientation constraint:** In the path of moving a component part, the orientation of part also imposes certain constraints. In fact, the concept of orientation constraints is relative as it depends largely upon the type of robot and the spatial relationship between component parts. When 'pick-and-place' operations are the operations to use, the orientation of assembling parts is along the  $z$  axis. After assembling  $P_a$  and  $P_b$  (Figure 3.3 (c)), part  $P1$  under  $P2$  must be assembled before  $P2$ . Consequently, the orientation constraint in this example is also denoted by  $PC(P1, P2)$ .

The constraints on geometry, stability, and orientation work fine with the 'pick-and-place' operations. However, other constraints will have to be introduced in order to bring robotic assembly systems closer to reality.



## **Chapter 4**

# **Graphic Robot Programming**

The basic goal in modern robot-based design is to make the teaching process as user-friendly as possible. Recent advances in computer graphics technologies have made it possible for robotic simulation. This chapter first gives a formal geometric definition to the robot assembly problem, then discusses the direct kinematics problem which determines the position and orientation of a robotic simulator from a given state of joints, and finally reaches to a definition of the ‘teach-in’ process. The objective of this chapter is to show how interactive graphics can be used in a virtual robot workcell.

### **4.1 Towards Off-line Programming**

There is a growing tendency in the research community towards semi-autonomous systems, where a given task would be mediated by a human, leading to the concept of shared autonomy.

### 4.1.1 Interactive Graphic Programming

While researchers are developing automated systems, interactive programming is one of the new topics that has recently attracted the attention of many researchers. The user of an interactive system works within a graphical robot workcell environment to program a robot performing specific tasks. Working with a virtual simulator greatly simplifies the programming of assembly tasks. The interactive capability of a system makes difficult tasks easy to conduct, for example, in assembly sequence specification and path planning. The system understands the assembly operations that an operator performs. And it can discard unnecessary computing which are often necessary with automatic programming. In such a sense, an interactive programming system is more dextrous than automatic programming systems. Figure 4.1 shows the structure of a graphic programming system, which outlines the interactive graphics approach of the thesis research.

### 4.1.2 Relation to Other Work

Efforts are currently underway to make robot easier to program. A major research activity aimed at the development of an off-line robot programming environment has been carried out in many places. For instance, HANDEY [64] is a vision-based system that is capable of planning for 'pick-and-place' operations. Several prototypes, using task-level programming methods, have been developed to ESPRIT 623 [45] and ROPSII [61]. They use task-level languages, instead of motions or other low-level robot related commands. Task-level systems, though useful, have not been completed

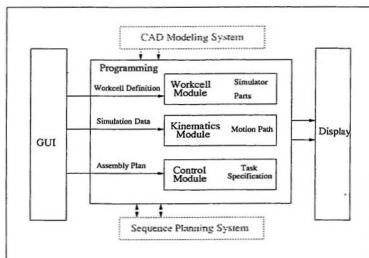


Figure 4.1: Structure of a graphic programming system for robotic assembly applications.

yet. Examples of interactive systems include the SMALL system [42] and the RV-M1 system [23], which provide a powerful graphics interface for easy-to-use robot programming. But these systems focus too much on graphical simulation and lack the ability to be integrated with the assembly planning process. Another prototype system RPD [15] has also been developed but it is more in the Programming by Demonstration (PbD) domain.

## 4.2 Geometric Robot Assembly Workcell

Generally speaking, a robot arm, a fixed station, and a set of workpieces are three basic components that composite a manufacturing workcell. The robot arm is required

to perform assembly tasks. The fixed station (workstation) is a tabletop or a wall. It supports the assembly (in the presence of gravity) and defines the boundary of a workcell. Workpieces are component parts of a product. In the thesis research, the shapes of the robot arm, the workstation, and the workpieces are assumed to consist of planar surfaces only. They can be represented by polyhedral objects.

### 4.2.1 A Simulated Robot

In robotics, only rigid bodies are considered. They represent mechanical links. These links are interconnected by joints. Eleven types of joints may be considered between two links [53], and two of them are used in the thesis research. They are revolutionary joints for rotational motions and sliding joints for translation. Figure 4.2 (a) shows a simulated robot arm operating with these two types of joints. It has a base support, an arm, and an end effector(gripper). The arm can be further divided to two individual sections, including an upper arm and a forearm. The segments between joints are called links. Links are the rigid members of a robot that support the loads carried by the robot. Joints in the example are either rotating, or sliding (or a combination of both). In particular, the simulated robot has three joints. The base is rotary; the upper arm moves horizontally; and the forearm moves vertically. A robot with this characteristic is called a cylindrical-coordinate robot. Since the cylindrical structure of robot can be modeled, the simulated robot is stored in the database of the system, together with all the other information that describes machines, workpieces and the cell layout.

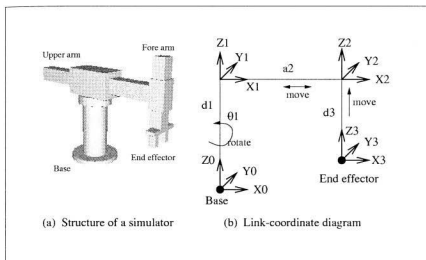


Figure 4.2: A robot simulator.

### 4.2.2 Object Modeling

The shapes and geometric relationships of the parts and robot arms are crucial in graphic assembly programming and planning. As polygon shapes are much less complex and can be rendered at higher rates than curved surfaces, they are used in the thesis to represent object models. For complex objects, they are constructed in forms of CSG structures [51, 11, 20] with a set of simple primitive objects by means of regularized Boolean set operators. Each of them is a tree with operations at the internal nodes and simple primitives at the leaves.

## 4.3 Robot Kinematics

Positioning the links of a robot involves finding a set of joint variables and link transformations for a particular manipulator goal point. This can be accomplished by either forward (or direct) kinematics, or inverse kinematics. While *direct kinematics* finds the position and orientation of a manipulator from the given states of all joints, *inverse kinematics* finds the states of all the joints from a given location and orientation of the manipulator. In general, inverse kinematics is much more difficult than forward kinematics because forward kinematics involves straightforward matrix multiplications, but inverse kinematics usually involves solving non-linear systems to obtain individual joint variables. More discussion on inverse kinematics can be found in [53, 49].

### 4.3.1 Homogeneous Transformations

To simulate the operation of a robot, a transformation from the Cartesian coordinate system to the robot coordinate system must be performed. The relative position and orientation between adjacent coordinate frames are determined by a homogeneous transformation. Mathematically, this transformation is represented by a  $4 \times 4$  matrix in the following format:

$$\mathbf{T} = \left[ \begin{array}{c|c} \begin{array}{c} \text{Rotation} \\ \text{matrix} \end{array} & \begin{array}{c} \text{Position} \\ \text{vector} \end{array} \\ \hline \begin{array}{c} \text{Perspective} \\ \text{transformation} \end{array} & \begin{array}{c} \text{Scaling} \\ \text{factor} \end{array} \end{array} \right] = \left[ \begin{array}{c|c} \begin{array}{c} \mathbf{R}_{3 \times 3} \\ \dots\dots\dots \\ \mathbf{f}_{1 \times 3} \end{array} & \begin{array}{c} \mathbf{P}_{3 \times 1} \\ \dots\dots\dots \\ \mathbf{W}_{1 \times 1} \end{array} \end{array} \right] \quad (4.1)$$

The upper-left  $3 \times 3$  submatrix describes the rotational relationship between the two coordinate systems. The upper-right  $3 \times 1$  column matrix describes the translation vector from the origin of the reference coordinate system to the origin of the rotated coordinate system. The lower-left  $1 \times 3$  row matrix provides the perspective transformation along the three coordinate axes. And the lower-right  $1 \times 1$  submatrix performs a scaling function. In manipulator kinematics, the elements of the perspective transformation matrix,  $f$ , are always set to zero ( $[000]$ ) and the scaling matrix,  $W$ , is always set to one ( $[1]$ ). Thus, Equation 4.1 can be simplified to Equation 4.2:

$$\mathbf{T} = \left[ \begin{array}{c|c} \mathbf{R}_{3 \times 3} & \mathbf{P}_{3 \times 1} \\ \hline \mathbf{0}_{1 \times 3} & \mathbf{1}_{1 \times 1} \end{array} \right] \quad (4.2)$$

### Transformations

The  $4 \times 4$  homogeneous transformation matrix  $\mathbf{T}$  in Equation 4.2 is the primary mathematical function used in robot kinematics to transform an arbitrary vector from a moving coordinate frame to a reference coordinate frame.

#### a. Translational Transformation

The position vector of a homogeneous transformation matrix tells the location of the origin of a link-attached coordinate frame when translated away from a reference coordinate frame. Let  $\mathbf{p} = p_x \mathbf{i} + p_y \mathbf{j} + p_z \mathbf{k}$  corresponds to this translation vector. A transformation matrix  $\mathbf{T}$  then takes the following form:

$$\mathbf{T} = \text{Trans}(p_x, p_y, p_z) = \begin{bmatrix} 1 & 0 & 0 & p_x \\ 0 & 1 & 0 & p_y \\ 0 & 0 & 1 & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (4.3)$$

### b. Rotation Transformation

The rotation matrix of a homogeneous transformation matrix describes the directions of the link-attached coordinate axes when rotated around an axis in the reference coordinate frame. Let  $x'-y'-z'$  be a coordination system rotated about the  $x$ -axis of the reference frame  $x-y-z$  by an angle  $\theta$  without moving its origin. The homogeneous transformation matrix is given by

$$\mathbf{T} = \text{Rot}(x, \theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (4.4)$$

Similarly, the homogeneous transformation matrices for rotation about the  $y$  and  $z$  axes are given by:

$$\mathbf{T} = \text{Rot}(y, \theta) = \begin{bmatrix} \cos \theta & 0 & \sin \theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (4.5)$$

and



$$\mathbf{T} = \text{Rot}(z, \theta) = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (4.6)$$

### 4.3.2 Arm Kinematic Equation

A direct kinematics solution starts with assigning the link-attached coordinate frame to each link of the manipulator. The link parameters between the  $i-1$ th and  $i$ th frames are used to establish the homogeneous transformation matrix  $\mathbf{T}_{i-1}^i$ . The state of the end effector, the  $n$ th link-attached frame, with respect to the base coordinate frame, frame 0, is found to be the chain products of the  $\mathbf{T}$  matrices. Let the  $n$ th frame be the coordinate system attached to the end effector and the 0th frame be the base coordinate system. Their spatial relationship is then represented by  $\mathbf{T}_0^n$ , which is a composition of  $n$  individual transformations

$$\mathbf{T}_0^n = \mathbf{T}_0^1 \mathbf{T}_1^2 \cdots \mathbf{T}_{n-1}^n \quad (4.7)$$

Figure 4.2 (b) shows a link-coordinate diagram for the simulator in Figure 4.2 (a). The simulator is an open kinematic chain consisting of three links  $L_i$  and three joints  $J_i$ , where  $i = 1, 2, 3$ . The state of the end effector, link-attached frame  $F_n$ , with respect to the base coordinate frame  $F_0$ , is determined by  $\mathbf{T}_{base}^{effector}$  as follows:

$$\mathbf{T}_{base}^{effector} = \mathbf{T}_0^3 = \mathbf{T}_0^1 \mathbf{T}_1^2 \mathbf{T}_2^3, \quad (4.8)$$

where  $\mathbf{T}_0^1 = Trans(0, 0, d1)Rot(z0, \theta1)$ ,  $\mathbf{T}_1^2 = Trans(a2, 0, 0)$ , and  $\mathbf{T}_2^3 = Trans(0, 0, -d3)$ .

## 4.4 Assembly Task Specification and Control

Robot programming and planning require a representation for the plans which not only are independent of the operating robot but also can be transformed into a representation that includes the robot. Such a representation should model the world components that can be connected and disconnected to define various aspects of the world. A programming environment can be decomposed into three distinct entities: tasks, a robot, and objects. Tasks are actions on objects; a robot is the performer of actions; and objects are the recipients of actions. Among these entities, 'action' is the common denominator. In a robotic environment, these actions represent physical motion. These are some primitive actions that cause a basic physical motion, such as rotation, locomotion, etc. Figure 4.3 lists the primitive actions employed in assembly task specification.

Task control is to interpret application programs and to supervise task execution. A graphical user interface has to be developed in such a way that an operator can efficiently control assembly tasks. There are different human-computer interaction functionalities that can be used in the specification of functional parameters. For

<b>Move</b>	<i>move along a plane to</i>
<b>Rotate</b>	<i>rotate around an axis</i>
<b>Pick</b>	<i>grasp and raise an object</i>
<b>Place</b>	<i>flip down (release) an object</i>

Figure 4.3: Primitive actions employed in assembly task specification

example, clicking a 'pick-up' button may be used to specify the 'grasp' of an object. And the motion of a robot can be controlled by a robot control box and several menus. A robot 'teach-in' box (processor), controlled with a keyboard, can be used to manually move robot joints, to record frame variables, and to specify trajectory segments. With a 'teach-in' box, the users can interactively specify sequences of tasks (the courses of actions) to program robot off-line. Given a robotic assembly  $\mathcal{A}(P, T)$  as in Figure 4.4, a sequence of tasks to accomplish the assembly is as follows:

AT 1: Fixing of P1 to Table.

AT 2: Fixing of P2 to Table.

AT 3: Assembly of P3 on P1 and P2.

AT 4: Assembly of P4 on P2.

In the programming process, each assembly task (AT) will be further refined to a sequence of elementary robot task operations (ERATO). For instance, the assembly task AT1 consists of the following ERATO sequence:

ERATO 1: Pick up P1.

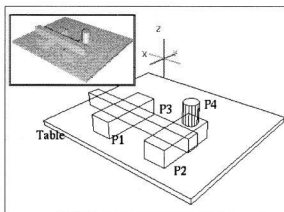


Figure 4.4: A robotic assembly scene for task specification.

**ERATO 2:** Move P1 to Table.

**ERATO 3:** Place P1 on Table with compliance.

## Chapter 5

### Robotic Assembly Sequencing

A feasible assembly sequence consists of a set of assembly operations that satisfy certain assembly constraints. The capability of an assembly system to find feasible assembly sequences is particularly useful in improving the efficiency of robotic assembly. It is not a new concept and has been discussed in both assembly planning and the design of object-level task programming languages. Instead of using the existing 'full-automatic' and 'editor' approaches, this chapter presents a novel method, called 'semi-automatic', for robotic assembly sequence planning [72]. This method uses the input module of off-line programming and thus significantly reduces the number of geometric reasoning steps. Presented in Section 5.1 is a model of robotic assembly sequencing that utilizes the concepts of assembly precedence graphs and feasible assembly trees. The strategy of 'semi-automatic' sequence planning is then illustrated with detail in Section 5.2.

## 5.1 A Model for Robotic Assembly Sequencing

Given a robotic assembly  $\mathcal{A}(\mathcal{P}, \mathcal{T})$ , since there are no two tasks applying on the same part, an ordered assembly tasks  $\tau_1 \tau_2 \dots \tau_n$  corresponds to an ordered sequence  $P_1^i P_2^i \dots P_n^i$  of a  $n$ -parts product. In the sequence,  $P_1^i$  is the first part to pick-and-place and  $P_n^i$  is the last. Therefore, a *robotic assembly* can also be defined in a simple way as follows:

**Definition 5.1 (Robotic Assembly)** *A robotic assembly is an assembly  $\mathcal{A}(\mathcal{P})$ , in which the product is assembled from  $n$  parts by a robot arm. An ordered set of parts  $P$  is assigned with  $n$  sequential ‘pick-and-place’ operation assembly tasks in a 3-dimensional workspace  $\mathbb{W}_A$  with no additional obstacles.  $\mathcal{P} = P_0^i P_1^i P_2^i \dots P_n^i$  is a feasible assembly sequence, where  $P_0$  denotes a stable workstation and  $\bigcup_{j=1}^n P_j^i = P = \{P_1, \dots, P_n\}$  is a set of parts to assemble.*

The above definition shows that the problem of robotic assembly sequencing is a manipulation planning problem, where the initial configuration satisfies the constraints in Definition 5.1 and the goal is to assemble a product from a set of distinct parts. Each assembly task defines a ‘pick-and-place’ operation. Motion planning however is not concerned because the paths of robot motions have been generated by an operator in off-line programming.

### 5.1.1 Precedence Constraints

An assembly plan consists of a set of assembly tasks with ordered constraints among its elements. These constraints are called *precedence constraints* that are used to find a valid order of assembly operations. They are caused by either the tasks themselves

or from other resources such as robot, gripper, etc. that execute the tasks. The order of assembly operations depends on the feasibility of assembly steps.

In fast planning, precedence constraints are further categorized into two different types: *direct precedence constraints* and *implicit precedence constraints*. Let  $PC$  denote a set of precedence constraints for a robot assembly  $\mathcal{A}(\mathcal{P})$  that has  $n$  parts, direct precedence constraints and implicit precedence constraints can be defined as follows.

**Definition 5.2 (Direct Precedence Constraints)** *Given a set of precedence constraints  $PC$ , a precedence constraint  $PC(P_i, P_j) \in PC$  is a direct precedence constraint if there is no  $P_k \in \mathcal{P}$  such that  $\exists PC(P_i, P_k) \in PC$  and  $\exists PC(P_k, P_j) \in PC$ .  $DPC$  then denotes a set of direct precedence constraints, i.e.,  $DPC \stackrel{\text{def}}{=} \{PC(P_i, P_j) | PC(P_i, P_j) \in PC \wedge \neg \exists P_k \in \mathcal{P} (PC(P_i, P_k) \in PC \wedge PC(P_k, P_j) \in PC)\}$ .*

Given any two parts, they have a direct precedence constraint if one part overlaps with the other one and there is no other parts in between them. Direct precedence constraints are expressed as explicit constraints. They are detected by applying geometric reasoning but not to be inferred from constraint propagation.

**Definition 5.3 (Implicit Precedence Constraints)** *Given a set of precedence constraints  $PC$ , a precedence constraint  $PC(P_i, P_j) \in PC$  is called an implicit precedence constraint if there exists  $P_k \in \mathcal{P}$  such that  $\exists PC(P_i, P_k) \in PC$  and  $\exists PC(P_k, P_j) \in PC$ .  $IPC$  then denotes a set of implicit precedence constraints, i.e.,  $IPC \stackrel{\text{def}}{=} \{PC(P_i, P_j) | PC(P_i, P_j) \in PC \wedge \exists P_k \in \mathcal{P} (PC(P_i, P_k) \in PC \wedge PC(P_k, P_j) \in PC)\}$ .*

Implicit precedence constraints represent the precedence relationships that can be inferred from explicit precedence constraints. For example, by applying constraint propagation,  $PC(i, j)$  can be deduced from  $PC(i, k)$  and  $PC(k, j)$ .

### 5.1.2 Robotic Assembly Sequence

Given a robotic assembly  $\mathcal{A}(\mathcal{P}, \mathcal{T})$  that has  $n$  parts, an ordered set of  $n$  robotic assembly tasks  $\tau_1 \tau_2 \dots \tau_n$  is an *assembly task sequence* if there are no two tasks applying on the same part. The output of the tasks is a product. For example, an assembly sequence that accomplished for the assembly in Figure 5.1 (a) is:

$$Table \xrightarrow{\tau_1} P1 \xrightarrow{\tau_2} P2 \xrightarrow{\tau_3} P3 \xrightarrow{\tau_4} P4 \quad (5.1)$$

An assembly sequence is said to be *feasible* if all its assembly tasks are geometrically and mechanically feasible, and the sub-assemblies of all tasks are stable. The assembly sequence in Equation 5.1 is feasible. An infeasible assembly sequence for the same assembly could be:

$$Table \xrightarrow{\tau_1} P1 \xrightarrow{\tau_2} P3 \xrightarrow{\tau_3} P2 \xrightarrow{\tau_4} P4 \quad (5.2)$$

It is infeasible because the third task  $\tau_3$  is not geometrically feasible. There is no collision free path to position  $P2$  onto the *Table* as in Figure 5.1 (a), once both  $P1$  and  $P3$  are joined into a subassembly.

### 5.1.3 Representation of Robotic Assembly Plan

There are many approaches for the representation of product assembly. They include representations based on directed graphs, on AND/OR graphs, on establishment conditions, and on precedence relationships [35, 37]. The following discussion presents



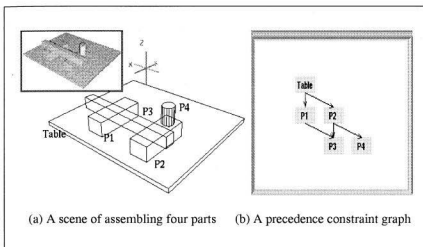


Figure 5.1: A sample robotic assembly and its constraint graph.

a method of the representation of robot's assembling a product based on a precedence constraint graph and an assembly tree.

### Precedence constraint representation

In the thesis research, precedence constraint graphs are used to represent robotic assemblies. A *precedence constraint graph* (CG) is an acyclic directed graph, in which the initial node represents a stable workstation, and the other nodes correspond to the set of assembly parts. An edge in a CG graph links two ordered nodes, and indicates the precedence relationship between them. In addition, a directed edge  $P_i \rightarrow P_j$  corresponds to a direct precedence constraint  $DPC(P_i, P_j) \in DPC$ .

**Definition 5.4 (Precedence Constraint Graph)** A *precedence constraint graph* of a robotic assembly  $\mathcal{A}(\mathcal{P})$  is an acyclic directed graph  $CG(\mathcal{P}, \mathcal{E})$ , in which each node of graph corresponds to a distinguished element of  $\mathcal{P}$ .  $\mathcal{E}$  is the set of  $n$  directed

edges. If  $E_{P_i, P_j}$  is a directed edge from  $P_i$  to  $P_j$ , then  $\mathcal{E} \stackrel{\text{def}}{=} \{E_{P_i, P_j} | \exists P_i \exists P_j (P_i, P_j \in \mathcal{P} \wedge DPC(P_i, P_j) \in DPC)\}$ .

Precedence constraint graphs are a convenient method to represent the flow of assembly operations. In a CG graph, the directed edges define the chronological order of assembly operations. For example,  $P_i \rightarrow P_j$  means that part  $P_j$  can only be assembled after the operation on another part  $P_i$  is completed. The information of parts and edges is stored in a constraint matrix CM. An example of precedence constraint graph is in Figure 5.1 (b), which represents a 4-parts assembly as in Figure 5.1 (a).

### Assembly tree representation

Since each assembly sequence is an ordered list, all the assembly sequences can be represented by a set of ordered lists, each of which corresponds to a different assembly sequence. As many assembly sequences have common subsequences, attempts have been made to create more compact representations that encompass all assembly sequences [37]. A *feasible assembly tree* (AT) is developed in the thesis to represent all feasible sequences of a robotic assembly.

**Definition 5.5 (Assembly Tree)** An assembly tree is a set of 3-tuples  $AT(\mathcal{P}, \mathcal{N}, \mathcal{E})$ . In the tree,  $\mathcal{P} = P_0 + P$ , where  $P_0$  is a stable workstation and  $P$  is the set of assembly parts. All the nodes of tree are in  $\mathcal{N} = \sum_{d=0}^n N^d$ , where  $N^d$  is a set of nodes whose depth in tree is  $d$ , and  $d \in \{0, \dots, n\}$  indicates the number of robotic assembly task that has been executed. The initial node  $N_0^0$  is associated with  $P_0$ , and every node  $N_i^d \in \mathcal{N} - \{N_0^0\}$  corresponds to a part  $P_j \in P$ .  $N_i^n$  is a tip node which stands for the last part to assemble. Moreover,  $\mathcal{E}$  denotes the set of directed edges, each of which corresponds a feasible robotic assembly task.

A feasible assembly tree is a tree with depth  $n$ . Each of its nodes has at most one parent. Each path of length  $n$  that walks through a sequence of nodes  $(N_i^0 N_i^1 \dots N_i^n)$  corresponds to a feasible assembly sequence. When a robotic assembly is represented with an assembly tree AT, finding a feasible assembly sequence becomes finding a path of length  $n$  in the tree. Consequently, finding all the feasible assembly sequences is simplified to generating an assembly tree for that assembly.

## 5.2 Automatic Sequence Planning

In practice it is difficult for the operator to visualize the concept of precedence constraint. It relies on an automatic approach to deduct assembly constraints and generate feasible sequences. Figure 5.2 shows the structure of a planning system for automatic assembly sequencing. The system consists of a reasoning module for detecting the geometric intersection of parts, a deduction module for deducting assembly precedence constraints, and a generation module for generating feasible assembly sequences. The following sections discuss the three modules in details.

### 5.2.1 Geometric Reasoning for Detecting Intersection of Parts

In order to automatically generate sequences, it is necessary to perform geometric reasoning in search of assembly precedence constraints. In the integrated model, geometric reasoning is only required to search DPC constraints. When a robot performs 'pick-and-place' operations, only the projection along the  $z$  axis is needed to check the feasibility of moving a mechanical part in a working environment. The projection

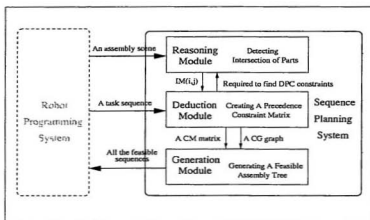


Figure 5.2: Structure of a planning system for automatic assembly sequencing.

plane perpendicular to the  $z$  axis is the  $XY$  plane. Therefore, in a 'pick-and-place' assembly system, geometric reasoning for determining the assembly precedence relation between two parts is simply to examine if their projections on the  $XY$  plane intersect.

Furthermore, with the restriction that all objects in the assembly space are merged from a set of polyhedral objects, the projection of an object on the  $XY$  plane is either a simply polygon or an arbitrary polygon. A *simple polygon*, called as a polygon for simplicity, is a primitive polygon without internal holes. On the contrary, an *arbitrary polygon* is constructed with a set of simple polygons by means of regularized Boolean set operators. For example, in the Figure 6.9(a), the project of the part *Spiece* on the  $XY$  plane is an arbitrary polygon that is composed from one large polygon subtracted other two small polygons. Here, without loss of generality, the thesis treats a cycle as a special polygon. Obviously, along with calculating the corresponding Boolean

set operators, a basic algorithm for checking the intersection of two simple polygons can examine whether the projections of two objects are intersection on a plane.

By notation, a polygon  $p$  is represented as a set of vertices (or points)  $\mathcal{V}^p$ .  $\mathcal{V}^p = \{\mathcal{V}_0^p, \dots, \mathcal{V}_{n^p}^p\}$ , where  $n^p$  is the number of vertices in the polygon  $p$ . An edge of  $p$  is a line  $\mathcal{L}_i^p$ , composed of its two end points  $(\mathcal{V}_{i-1}^p, \mathcal{V}_i^p)$ . One way to check the intersection of two polygons is to apply the 'scan-line' algorithm. This method checks if a point on the scan-line is inside both of the two polygons. This scan-line method is useful to check the intersection of two arbitrary shapes. But, to the problem of the intersection of two polygons, this method is computationally expensive. The following algorithm uses geometric reasoning, and is more efficient than the scan-line method to check the intersection of two polygons.

#### Algorithm 5.1 (Intersection of polygons)

**Input:** Two polygons  $p_1$  and  $p_2$ , and they have been constructed with  $n^{p_1}$  vertices and  $n^{p_2}$  vertices respectively, where  $n^{p_1}, n^{p_2} \geq 3$ .

**Output:** 1, if they intersect, or 0, if they don't intersect.

**Description:** It is easy to observe that  $p_1$  and  $p_2$  intersect if and only if at least one of the following conditions is satisfied.

1. Do the boundaries of  $p_1$  and  $p_2$  intersect?
2. Is there a vertex of  $p_1$  in  $p_2$ ?
3. Is there a vertex of  $p_2$  in  $p_1$ ?

**Procedure:** for each  $\mathcal{L}_i^{p_1}, i \in [0, n^{p_1}]$   
           for each  $\mathcal{L}_j^{p_2}, j \in [0, n^{p_2}]$   
               if (  $\mathcal{L}_i^{p_1}$  intersect  $\mathcal{L}_j^{p_2}$  )  
                   Return 1 ( the boundaries of  $p_1$  and  $p_2$  intersect );

$TimeIntsect \leftarrow 0$  ;  
 take a point  $\mathcal{V}_{int}^{p_1}$  in  $p_1$ ;

```

make a line  $\mathcal{L}_{md}$  connecting two vertices  $\mathcal{V}_{int}^{p_1}$  and  $\mathcal{V}_k^{p_2}$ ,  $\mathcal{V}_k^{p_2} \in \mathcal{V}^{p_2}$ ;
for each  $\mathcal{L}_i^{p_1}$ ,  $i \in [0, n^{p_1}]$ 
    if ( $\mathcal{L}_i^{p_1}$  intersect  $\mathcal{L}_{md}$  )
         $TimeIntsect \leftarrow TimeIntsect + 1$  ;
If (  $TimeIntsect$  is an even number )
    Return 1 (  $p_2$  lies in  $p_1$  );

 $TimeIntsect \leftarrow 0$  ;
take a point  $\mathcal{V}_{int}^{p_2}$  in  $p_2$ ;
make a line  $\mathcal{L}_{md}$  connecting two vertices  $\mathcal{V}_{int}^{p_2}$  and  $\mathcal{V}_k^{p_1}$ ,  $\mathcal{V}_k^{p_1} \in \mathcal{V}^{p_1}$ ;
for each  $\mathcal{L}_i^{p_2}$ ,  $i \in [0, n^{p_2}]$ 
    if ( $\mathcal{L}_i^{p_2}$  intersect  $\mathcal{L}_{md}$  )
         $TimeIntsect \leftarrow TimeIntsect + 1$  ;
If (  $TimeIntsect$  is an even number )
    Return 1 (  $p_1$  lies in  $p_2$  );
Return 0 (  $p_1$  and  $p_2$  don't intersect);
end; {Procedure}

```

Algorithm 5.1 is a basic algorithm for checking the intersection of two *simple* polygons. For a polygon  $p_1$  with  $n^{p_1}$  vertices and a polygon  $p_2$  with  $n^{p_2}$  vertices, the time bound of checking their intersection is  $O(n^{p_1}n^{p_2})$ . To check the intersection of two polygons which include an *arbitrary* polygon, the planner then requires to calculate the Boolean set operators along with this basic algorithm.

In the geometric reasoning process, the planner uses an *intersection matrix* ( $IM$ ) to store the information of intersection obtained from Algorithm 5.1. An element  $IM(i, j)$  of 1 or 0, where  $0 \leq i, j \leq n$ , indicates the intersection or non-intersection status between the  $i$ th and  $j$ th parts respectively. Similarly,  $IM(i, j)$  shows an implicit relationship when marked with -1 or an invalid relationship when marked with -9. An implicit relationship between two parts means that this relationship can be

inferred from *IM* elements, rather than being directly obtained from geometric reasoning. To a sequence planning process, the number of its geometric reasoning checks is equal to the total number of the positive elements in its *IM* matrix because the geometric reasoning is only required to determine these elements. At the left side of Figure 5.3 is the intersection matrix for the sample assembly in Figure 5.1, and it shows the intersection relationships between the parts and the table.

IM	Table	P1	P2	P3	P4
Table	-9	-1	-1	-1	-1
P1	-9	-9	0	1	0
P2	-9	-9	-9	1	1
P3	-9	-9	-9	-9	0
P4	-9	-9	-9	-9	-9

CM	Table	P1	P2	P3	P4
Table	-9	3	3	2	2
P1	-9	-9	0	3	0
P2	-9	-9	-9	3	3
P3	-9	-9	-9	-9	0
P4	-9	-9	-9	-9	-9

Figure 5.3: *IM* matrix and *CM* matrix of the sample robotic assembly

### 5.2.2 Deduction of Assembly Precedence Constraints

The planning process uses another matrix, namely a *constraint matrix* (*CM*), to specify precedence constraints. The rows and columns of its elements  $CM(i, j)$ ,  $0 \leq i, j \leq n$ , are ordered by an initial feasible assembly sequence. An entry  $CM(i, j)$  represents the precedence constraint relationship between the  $i$ th and  $j$ th parts.  $CM(i, j)$  is 0 when there is no precedence constraint between them. When  $CM(i, j)$  is marked with -9, it indicates a situation that can never take place in the current model. If, however,  $CM(i, j)$  is marked with 3 or 2, it indicates that either there exists a precedence constraint relation between the  $i$ th and the  $j$ th parts or part  $P_j$  can only be

assembled after part  $P_i$ . The former is a *direct precedence constraint* and the latter is an *implicit precedence constraint*. At the right side of Figure 5.3 is the precedence constraint matrix for the sample assembly in Figure 5.1.

A constraint matrix keeps all the information about what is obtainable from both the symbolic and geometric reasonings. Geometric reasoning is useful in finding direct precedence constraints. Symbolic reasoning, on the other hand, can be used to search for implicit precedence constraints. Symbolic reasoning has higher priority than geometric reasoning because it is faster. Only when symbolic reasoning fails to find the precedence constraints will the geometric reasoning be used. Searching precedence constraints starts with re-using an initial feasible assembly sequence, which is directly obtained from the analysis of an ordered robot action in off-line programming. So, instead of using full checks in 'full-automatic' planning systems, it only requires partial checks within the upper-right half of a CM matrix. Besides, most of the robotic assembly operations perform 'face-contact' operations, in which one part is assembled onto another (Figure 6.6 (a)). Therefore, a more promising way of searching is to select an element  $CM(i, j)$  on the diagonal line in a CM. An algorithm for automatically detecting precedence constraints is presented below.

#### Algorithm 5.2 (Creation of Precedence Constraint Matrix)

- Input:** An intersection matrix IM, obtained from geometric reasoning.  
**Output:** a precedence constraint matrix CM.  
**Description:** For every  $CM(P_i, P_j)$  on a diagonal line in the upper-right half of CM, search for an implicit precedence constraint between  $P_i$  and  $P_j$ . If there is not such implicit precedence constraint, geometric reasoning is performed to determine a direct precedence constraint based on their projection intersection. The row of elements for the table



is also determined according to the precedence constraints between  $Tab$  and  $P_i$ .

```

Procedure: for each  $CM(P_i, P_j) \leftarrow -9$ , where  $i \in [0, n], j \in [0, n]$ ;
for each  $i \in [0, n-1]$ 
     $j \leftarrow 0$ ;
    for each  $g \in [i+1, n]$ 
         $j = j + 1$ ;
        if  $CM(P_j, P_g) < 0$  (search for implicit constraint)
            for each  $k \in [j+1, g-1]$ 
                if  $CM(P_j, P_k) + CM(P_k, P_g) > 3$ 
                     $CM(P_j, P_g) = 2$ ;
                    break;

            if  $CM(P_j, P_g) < 0$  (search for direct constraint)
                 $CM(P_j, P_g) = 3 \times IM(P_j, P_g)$ ;

     $CM(Tab, P_i) \leftarrow 3$ ;
    for each  $j \in [1, i]$ 
        if  $CM(P_j, P_{i+1}) \geq 2$  (find  $P_{i+1}$  on  $P_j$ )
             $CM(Tab, P_{i+1}) = 2$ ;
end; {Procedure}

```

Algorithm 5.2 is presented to enrich the communication between the assembly sequencer (Algorithm 5.3) and the geometric reasoner (Algorithm 5.1). The number of calls to the geometric reasoner in Algorithm 5.2 is the overriding factor for the total running time of the automatic sequence planning. Using the concept of a IM matrix, the number of calls to the geometric reasoner will be easily computed for a  $n$ -parts assembly. Consider the worst situation where there is no implicit precedence constraint between the assembled parts, such as in Figure 6.7(a), the geometric reasoning is required to determine all of the elements  $IM(i, j)$ , where  $1 \leq i \leq n-1$  and  $i+1 \leq j \leq n$ . Thus, in the worst case, the number of calls to the geometric reasoner

$$= \sum_{i=0}^{n-1} i = n(n-1)/2 = O(n^2).$$

From the constraint matrix CM of a robotic assembly  $\mathcal{A}(\mathcal{P})$ , its constraint graph can be easily created. The first step is to create an initial node for the table. Underneath the *Table* node is a list of  $n$  nodes representing all the component parts. Then, for  $0 \leq i, j \leq n$ , there is a directed edge from a node  $P_i$  to another node  $P_j$  if  $CM(P_i, P_j) = 3$ . Figure 5.1(b) shows the constraint graph obtained from a constraint matrix CM shown in the right side of Figure 5.3.

### 5.2.3 Generation of Feasible Sequences

From a constraint matrix or constraint graph, all the feasible assembly sequences can be generated in forms of an assembly tree. For example, the assembly tree in Figure 5.4 is generated from the constraint graph in Figure 5.1 (b). It starts with the *Tab* node (Figure 5.4 (a)), and then expands to include nodes  $P1$  and  $P2$  as its two successors in the tree (Figure 5.4 (b)) since  $P1$  and  $P2$  are the two direct children of *Tab* in the graph (Figure 5.1 (b)). To further expand node  $P1$ , its brother node  $P2$  is directly placed to the next level in the tree (Figure 5.4 (c)). However, node  $P3$ , which is a child of  $P1$  in the graph (Figure 5.1 (b)), has to be abandoned since  $P2$  and  $P3$  have a precedence restriction in the graph. Otherwise, a child node in the graph become a child node in the tree if there are not precedence constraints attached, as in the case of  $P4$  for expanding nodes at  $P2$  (Figure 5.4 (c)). The tree construction is finished when each path contains all the parts. Figure 5.4 (e) shows the generation of the assembly tree, where its nodes are numbered in the order of their expansion. The tree is finished with a breadth-first search because the expansion of nodes in the tree

proceeds along a “contours” of equal depth. But it may also be a deep-first searching. A general algorithm of generating a feasible assembly tree from a constraint matrix is given below.

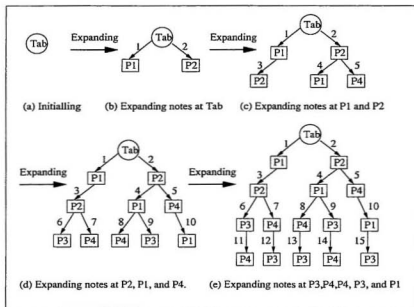


Figure 5.4: Generating an assembly tree for the sample robot assembly.

### Algorithm 5.3 (Generation of feasible assembly tree AT)

**Input:** A precedence constraint matrix CM.

**Output:** A feasible assembly tree AT.

**Description:** The steps of generating an assembly tree AT is as follows:

1. Choose the root node  $Tab$  as the current expanding node  $N_i^d$  for expansion, where  $i = 0, d = 0$ .
2. Select the sibling (brother) nodes of  $N_i^d$ , i.e.,  $S^{N_i^d}$ , with respect to the expanding subtree as its new children for expansion.

3. Search for the parts that have direct precedence constraints with the expanding node  $N_i^d$  and do not have precedence constraints with the sibling nodes of  $N_i^d$ , and expand them as its additional children.
4. Take one by one the outgoing nodes as the current expanding node  $N_i^d$ , and then go to Step 2 for the next round of expansion.

Procedure: CreateFeasibleAssemblyTree

```

 $N_1^0 = Tab; N^1 \leftarrow \emptyset;$ 
PROCEDURE CreateChildOfNode ( $N_i^0$ );
for each  $d \in [1, n-1]$ 
   $N^{d+1} \leftarrow \emptyset;$ 
  for each  $N_i^d \in N^d$ , where  $N^d$  is a set
    of the nodes whose depth are  $d$  in the tree.
    PROCEDURE CreateChildOfNode ( $N_i^d$ );
end; {Procedure}

```

Procedure: CreateChildOfNode ( $N_i^d$ )

```

 $Kid^{N_i^d} \leftarrow \emptyset;$ 
 $S^{N_i^d} \leftarrow$  a set of the sibling nodes of  $N_i^d$ ;
for each  $P_j \in \{P_j | P_j \in P [CM(N_i^d, P_j) =$ 
   $3 \wedge \exists S_k^{N_i^d} \in S^{N_i^d} (CM(S_k^{N_i^d}, P_j) \neq 2 \text{ or } 3)]\}$ 
   $Kid^{N_i^d} = Kid^{N_i^d} \cup P_j;$ 
 $Kid^{N_i^d} = S^{N_i^d} \cup Kid^{N_i^d};$ 
return  $N_i^{d+1} = N_i^{d+1} \cup Kid^{N_i^d};$ 
end; {Procedure}

```

Because of the complex ways in which the geometry of an assembly can affect the size of its assembly tree AT, it is difficult to find meaningful bounds on the computation required to build it. For instance, given an assembly with  $n$  parts, the number of nodes in an assembly tree AT can range from  $n+1$  when there is only

one legal sequence (as in Figure 6.6), to  $\sum_{i=1}^n (\prod_{j=n+1-i}^n j) + 1$  when all sequences are legal (as in Figure 6.7). Consider the situation in which all sequences of assembly are valid, such as in Figure 6.7, the total number of assembly sequences is  $n!$ . Therefore, the complexity of the presented assembly sequencing can be a NP-complete problem.

## **Chapter 6**

### **Implementation and Examples**

To illustrate the operation of the assembly system developed in this thesis, a prototype system called INTEG has been designed and implemented. A complete description of the development of the whole system could be rather lengthy as it requires discussions on GUI design with C/C++ programming and X-toolkits on Unix systems. For simplicity, Section 6.1 gives a brief overview on the current system INTEG with an emphasis on its functionality and its interface to the users. A set of experiment examples follows in Section 6.2.

#### **6.1 A Prototype INTEG**

A prototype is useful for the discovery of system specifications and helps to reduce the overall development cost [19]. To illustrate the integrated approach of robot assembly programming and sequence planning, a prototype system called INTEG has been designed and developed. A proof-of-concept implementation of basic procedures has been completed with the C/C++ programming language and the SRGP/SPHIG

graphics toolkits [20]. C/C++ programming language is used to define data structures and to implement algorithms, while SRGP/SPHIG toolkits is used to build a graphics interface and to display simulations. The *INTEG* currently runs under Unix/X-window and Sun-OS environments.

### 6.1.1 Structure of *INTEG*

*INTEG* prototype is a CAD/CAM tool for robot-based assembly automation. Its completed system should eventually integrate model designing, robot programming, and sequence planning processes with a common database. Figure 6.1 shows the architecture of *INTEG*, which is composed of several subsystems. They are the modeling, programming, and sequencing subsystems. The modeling subsystem handles object modeling, object selection, and the specification of functional parameters for workpieces arrangement. The programming subsystem allows the user to define a workstation and a robot, to specify assembly tasks, and to control robot paths within a graphics interface. And the sequencing subsystem provides symbolic reasoning and geometric reasoning. It deducts assembly precedence relations, creates precedence constraint graphs, and generates all the feasible assembly sequences for robotic assembly tasks. In the *INTEG* system, diversified expertise can be integrated to improve the flow of information among design, programming, and sequencing. For example, the information generated in the sequencing subsystem can be fed back to the programming subsystem for re-programming or to the modeling subsystem for the DFA/DFM (Design for Assembly and Design for Manufacturing [46, 30]) task analysis.

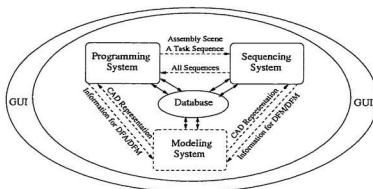


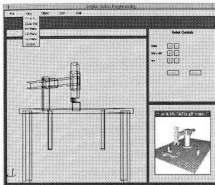
Figure 6.1: The architecture of INTEG.

### 6.1.2 Interface Controls and Functions

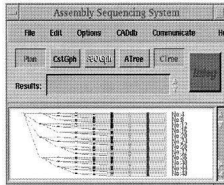
A proof-of-concept implementation of basic procedures focuses on the development of programming and sequencing subsystems. In the programming subsystem, the basic layout of the graphics interface is divided into four main regions, as shown in Figure 6.2 (a). The main portion of the screen is taken up by a viewing window, which displays the virtual robot-based assembly scene. On the top is a set of buttons for function specification, which, when being clicked upon, pop-up submenus for users to select items such as “file”, “view”, “color”, “styles”, “select”, and “help”. The upper-right is the ‘teach-in’ box area for path control and task specification. The lower right of the screen opens up a window for real-time robot simulation. The robot simulator can be zoomed, viewed, and controlled in the main window. Figure 6.3 shows an example of multiple views of the simulator, which is also provided within the main window. In the sequencing subsystem, the interface consists of three regions, as shown in Figure 6.2 (b). The upper region, which displays a set of menu-buttons and



control-buttons, allows the input of commands and parameters. The lower region, on the other hand, displays sequencing results such as CG graphs, AT graphs, and feasible sequences. The small region in between is a message window for feedback messages.



(a) Programming system



(b) Sequencing system

Figure 6.2: A screen display of programming and sequencing systems.

## 6.2 Experiment Examples

The following three sections demonstrate the implementation results of the assembly system INTEG. Section 6.2.1 gives a robotic assembly task to assemble eight randomly placed disjoint parts. To evaluate the performance, Section 6.2.2 gives the best case and worst case for the task of assembling seven simple parts. Section 6.2.3 exercises the system with the pendulum assembly task, which is a widely tested example in robotics research and application.

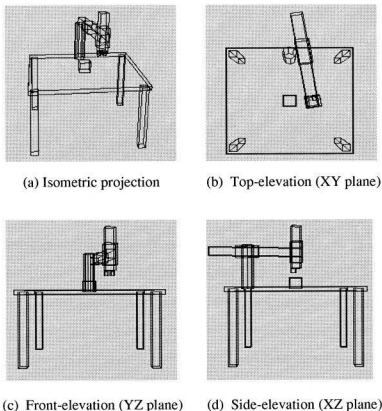


Figure 6.3: Multiple views of the simulator.

### 6.2.1 An Assembly of Eight Random Parts

An assembly instance can be represented unambiguously by a collection of solid models (i.e., unambiguous representations of rigid solids), each of which has a geometric transformation that defines its position in a workcell. Figure 6.4 (a) illustrates an assembly of eight randomly placed parts. The parts are assembled in the programming subsystem in an order given below.

**AssTask 1:** Fixing of P1 to Table.

**AssTask 2:** Fixing of P2 to Table.

**AssTask 3:** Fixing of P3 to Table.

**AssTask 4:** Fixing of P4 to Table.

**AssTask 5:** Assembly of P5 on P4 and P3.

**AssTask 6:** Assembly of P6 on P5 and P2.

**AssTask 7:** Assembly of P7 on P6 and P1.

**AssTask 8:** Assembly of P8 on P7.

The precedence constraint graph and feasible assembly sequences are generated by the sequencing subsystem. In the sequence planning, the system first generates the intersection matrix and the precedence matrix, and then the precedence constraint graph and a feasible assembly tree. Figure 6.5 shows the IM matrix and the CM matrix for this 8-parts assembly. Figure 6.4 (b) and Figure 6.4 (c) show the precedence constraint graph and the feasible assembly tree respectively. The information gained from the test shows that the total number of feasible assembly sequences is 48, which requires only 16 geometric reasoning checks for the deduction of assembly precedence constraints. The geometric reasoning checks are only performed on those positive elements in the intersection matrix IM, as shown in the left side of Figure 6.5.

### 6.2.2 Two Special Assembly Cases

To evaluate the performance, the INTEG system has been tested in several special cases. One of them is the best case of assembling seven simple parts, such as in

Figure 6.6. Within this example, seven geometric reasoning checks are used to deduct assembly constraints. The total number of feasible assembly sequences is one only. In the worst case of assembling seven parts, shown in Figure 6.7, 21 geometric reasoning checks are used to deduct assembly constraints and the total number of feasible assembly sequences reaches to 5,040.

### 6.2.3 Pendulum Assembly Task

One typical example of robotic assembly is the Pendulum Assembly Task [3]. It was evolved from the Cranfield Assembly Benchmark Kit [32, 45, 68] (Figure 6.8(b)) by reducing the 18 involved parts to nine (Figure 6.9(a)). The assembly benchmark shown in Figure 6.8 is a famous robotic assembly example that was used by Cranfield Institute of Technology (England) for on-line testing of robot programming. An assembly sequence of programming the nine-parts Pendulum task off-line with INTEG could be  $\tau_1\tau_2\tau_3\tau_4 \dots \tau_8\tau_9$ , where:

- $\tau_1$ : Fixing of Splate1 into table.
- $\tau_2$ : Insertion of Spacer1 into Splate1.
- $\tau_3$ : Insertion of Spacer2 into Splate1.
- $\tau_4$ : Insertion of Spacer3 into Splate1.
- $\tau_5$ : Insertion of Spacer4 into Splate1.
- $\tau_6$ : Assembly of Shaft on Splate1.
- $\tau_7$ : Insertion of Spiece into Spacer1 and Spacer2.
- $\tau_8$ : Assembly of Lever on Shaft.
- $\tau_9$ : Assembly of Splate2.

Its corresponding feasible assembly sequence is

$$\begin{aligned} &Table \xrightarrow{T_1} Splate1 \xrightarrow{T_2} Spacer1 \xrightarrow{T_3} Spacer2 \xrightarrow{T_4} Spacer3 \xrightarrow{T_5} \\ &Spacer4 \xrightarrow{T_6} Shaft \xrightarrow{T_7} Spiece \xrightarrow{T_8} Lever \xrightarrow{T_9} Splate2, \end{aligned}$$

or

$$Table \xrightarrow{T_1} P1 \xrightarrow{T_2} P2 \xrightarrow{T_3} P3 \xrightarrow{T_4} P4 \xrightarrow{T_5} P5 \xrightarrow{T_6} P6 \xrightarrow{T_7} P7 \xrightarrow{T_8} P8 \xrightarrow{T_9} P9.$$

In the sequence,  $P1$  is for  $Splate1$ ,  $P2$  for  $Spacer1$ ,  $P3$  for  $Spacer2$ ,  $P4$  for  $Spacer3$ ,  $P5$  for  $Spacer4$ ,  $P6$  for  $Shaft$ ,  $P7$  for  $Spiece$ ,  $P8$  for  $Lever$ , and  $P9$  for  $Splate2$ . In sequence planning, the geometric intersection matrix and precedence matrix are first created; then the precedence graph and feasible assembly sequences are generated from the CM matrix. Figure 6.10 shows the IM matrix and the CM matrix for the pendulum assembly; and Figure 6.9 (b) shows its precedence graph. A full assembly tree and all the feasible sequences for pendulum assembly are provided in detail in Appendix. The resulting information gained from this experiment shows that the total number of feasible assembly sequences is 840, during which only 30 geometric reasoning checks are needed for constraint deduction. The geometric reasoning checks are only required to determine those positive elements in the intersection matrix IM, as shown in the left side of Figure 6.10.

### 6.2.4 On Optimal Planning and Re-programming

The thesis does not explicitly discuss on optimal planning and re-programming issues. This is because the thesis research focuses on finding all the feasible solutions first.

The users then use this general solution to solve their own special interesting problems with applying the specific extrinsic constraints. Normally, there are two approaches of finding optimal sequences. One, qualitative approach, is to develop rules to eliminate assembly plans that include difficult tasks or awkward intermediate sub-assemblies. Another quantitative approach is to introduce an evaluation to compute the merit of assembly plans. It may be based on the cost of the resources needed to complete an assembly, on the total time required, or on the difficulty of execution. Concerning assembly stability here, we have a scheme that assembling large part has higher priority than assembling small one. Then the optimal assembly sequence of Pendulum assembly (shown in Figure 6.9), for instance, can be searched from the assembly tree and found as follows.

$$\begin{aligned} &Table \xrightarrow{\tau_1} Splate1 \xrightarrow{\tau_6} Shaft \xrightarrow{\tau_8} Lever \xrightarrow{\tau_2} Spacer1 \xrightarrow{\tau_3} \\ &Spacer2 \xrightarrow{\tau_7} Spiece \xrightarrow{\tau_4} Spacer3 \xrightarrow{\tau_5} Spacer4 \xrightarrow{\tau_9} Splate2 \end{aligned}$$

Furthermore, consider a fixed set of robotic assembly task with only their order needing to be changed to perform a re-program of robot, the ordered robotic assembly tasks  $\tau_1\tau_6\tau_8\tau_2\tau_3\tau_7\tau_4\tau_5\tau_9$  can be directly used for automatically re-programming a robot.

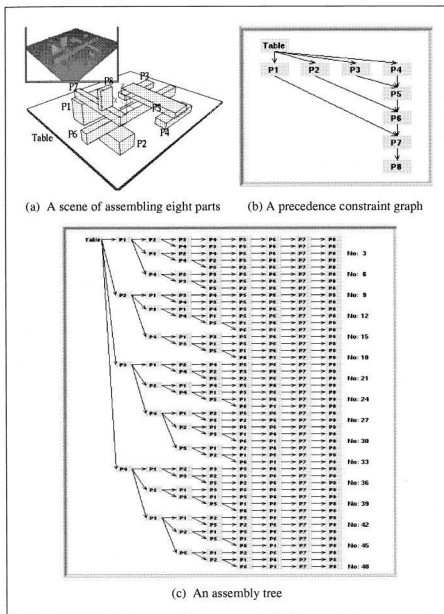


Figure 6.4: An assembly of eight random parts.

IM	Tab	P1	P2	P3	P4	P5	P6	P7	P8
Tab	-9	-1	-1	-1	-1	-1	-1	-1	-1
P1	-9	-9	0	0	0	0	0	1	-1
P2	-9	-9	-9	0	0	0	1	-1	-1
P3	-9	-9	-9	-9	0	1	-1	-1	-1
P4	-9	-9	-9	-9	-9	1	-1	-1	-1
P5	-9	-9	-9	-9	-9	-9	1	-1	-1
P6	-9	-9	-9	-9	-9	-9	-9	1	-1
P7	-9	-9	-9	-9	-9	-9	-9	-9	1
P8	-9	-9	-9	-9	-9	-9	-9	-9	-9

CM	Tab	P1	P2	P3	P4	P5	P6	P7	P8
Tab	-9	3	3	3	3	2	2	2	2
P1	-9	-9	0	0	0	0	0	3	2
P2	-9	-9	-9	0	0	0	3	2	2
P3	-9	-9	-9	-9	0	3	2	2	2
P4	-9	-9	-9	-9	-9	3	2	2	2
P5	-9	-9	-9	-9	-9	-9	3	2	2
P6	-9	-9	-9	-9	-9	-9	-9	3	2
P7	-9	-9	-9	-9	-9	-9	-9	-9	3
P8	-9	-9	-9	-9	-9	-9	-9	-9	-9

Figure 6.5: IM matrix and CM matrix for the 8-parts assembly task



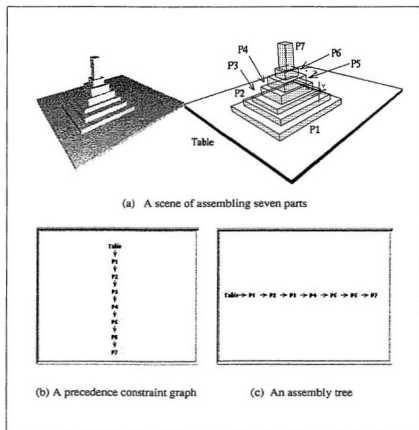


Figure 6.6: A simple assembly case for assembling seven parts.

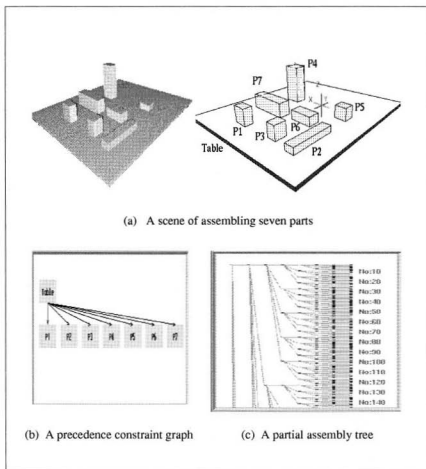
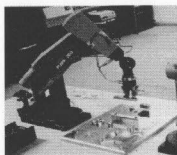
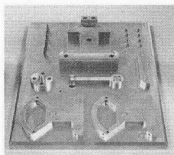


Figure 6.7: A worst assembly case for assembling seven parts.



(a) Workcell for CABK assembly



(b) 18-parts CABK assembly

Figure 6.8: A robotic assembly for Cranfield Assembly Benchmark Kit.

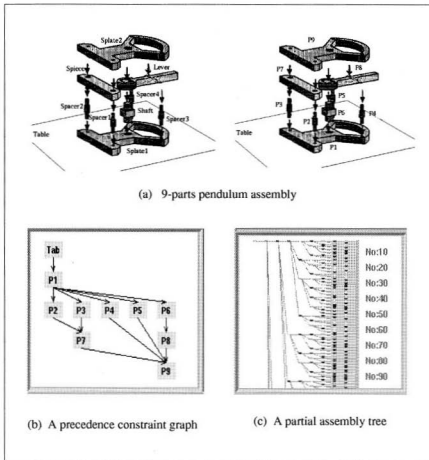


Figure 6.9: A robotic assembly for Pendulum Assembly.

IM	$T_{ub}$	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	$P_6$	$P_7$	$P_8$	$P_9$
$T_{ub}$	-9	-9	-9	-9	-9	-9	-9	-9	-9	-9
$P_1$	-9	-9	1	1	1	1	1	-1	-1	-1
$P_2$	-9	-9	-9	0	0	0	0	1	0	-1
$P_3$	-9	-9	-9	-9	0	0	0	1	0	-1
$P_4$	-9	-9	-9	-9	-9	0	0	0	0	1
$P_5$	-9	-9	-9	-9	-9	-9	0	0	0	1
$P_6$	-9	-9	-9	-9	-9	-9	-9	0	1	-1
$P_7$	-9	-9	-9	-9	-9	-9	-9	-9	0	1
$P_8$	-9	-9	-9	-9	-9	-9	-9	-9	-9	1
$P_9$	-9	-9	-9	-9	-9	-9	-9	-9	-9	-9

CM	$T_{ub}$	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	$P_6$	$P_7$	$P_8$	$P_9$
$T_{ub}$	-9	3	2	2	2	2	2	2	2	2
$P_1$	-9	-9	3	3	3	3	3	2	2	2
$P_2$	-9	-9	-9	0	0	0	0	3	0	2
$P_3$	-9	-9	-9	-9	0	0	0	3	0	2
$P_4$	-9	-9	-9	-9	-9	0	0	0	0	3
$P_5$	-9	-9	-9	-9	-9	-9	0	0	0	3
$P_6$	-9	-9	-9	-9	-9	-9	-9	0	3	2
$P_7$	-9	-9	-9	-9	-9	-9	-9	-9	0	3
$P_8$	-9	-9	-9	-9	-9	-9	-9	-9	-9	3
$P_9$	-9	-9	-9	-9	-9	-9	-9	-9	-9	-9

Figure 6.10: IM matrix and CM matrix for the pendulum assembly task.

## **Chapter 7**

### **Conclusions**

This thesis presents an integrated approach to robotic assembly programming and sequence planning for the development of robot-based assembly automation systems. This approach recognizes the importance of off-line programming techniques in robot assembly. By applying off-line programming methods as a sequence planner's input modus, it develops an integrated model to automatically generate different robotic assembly sequences and explores the feature of 'automatic re-programming'. The validity of this integrated approach is justified with a prototype system INTEG. Two of the INTEG's processes have been implemented. They are the programming process and the sequence planning process. The main research contributions of this thesis research are summarized in Section 7.1. Its anticipated impact on assembly automation is given in Section 7.2. Future research is also discussed in Section 7.3.

## 7.1 Research Contributions

Off-line programming and assembly sequence planning are two different research areas that have been addressed by many researchers. However, few attempts have been devoted to a tight integration of both the areas. Most research approaches assume that the sequence planning process finishes before executing the off-line programming process, and therefore deals with the two processes separately. The related work rests either in assembly sequence planning, where the manipulator constraints or the virtual process of generating assembly sequences are not concerned [5, 47, 35, 50, 45], or in off-line robot programming, where the functions of assembly sequence planning are ignored [34, 23, 26, 12, 62]. In these systems, programming and sequencing modules are loosely coupled.

The integrated approach presented in this thesis overcomes the problems of isolated approaches. Its advantages come mainly from the following two aspects.

- **Dextrous and Intelligent Programming**

The interactive graphics capability enables the system to understand the assembly operations that an operator performs. It, for instance, avoids unnecessary computations which are often required with an automatic programming system. The system also allows the visualization of assembly operation processes and sequence planning processes. Therefore, this integrated approach is more dextrous than isolated approaches. With a sequence planner integrated in the system, feasible assembly sequences can be automatically generated and directly used to re-program robots to achieve higher efficiency. This 'automatic

re-programming' feature is one of main capabilities that an intelligent robotic assembly system should have.

- **Automatic and Efficient Sequencing**

Since the results of off-line programming are used as an input to the assembly planner in the integrated system, the planner can directly retrieve necessary information from the programming subsystem and therefore performs sequence planning without human intervention. In addition, this integrated approach significantly reduces the number of geometric reasoning steps by utilizing precedence constraint expressions such as an ordered robot assembly task sequence. Evaluating the precedence expressions and executing partial geometric checks are usually much faster than performing full geometric checks. Therefore, this integrated approach offers significant computational advantages over those 'full-automatic' planning approaches. It is more efficient to conduct geometric reasoning with the INTEG system.

## 7.2 Anticipated Impact

This research explores a way to integrate off-line programming with sequence planning for the ultimate goal of robot-based assembly automation. It helps improving the productivity of operators by allowing them to program assembly tasks through a graphic interface and providing them with automatic re-programming. The planning algorithms developed in the thesis are also useful in other planning and scheduling applications, including VLSI design, high-volume repetitive handling and assembling,



autonomous robot navigation, and work-flow control.

The research results, presented in the previous chapters lead to a new paradigm for the development and practical application of virtual robot-based systems in mechanical assembly automation. These systems will allow the users to program robots off-line for assembly tasks, to derive different solutions for productivity and manufacturability analysis, to select an optimal sequence for 'automatic re-programming', and to instruct robots to follow the computed paths and perform assembly tasks on-line.

The thesis research is an effort of systematically developing design, planning, and programming processes for robot-based assembly automation. The experience gained in the development of the prototyping system can be used to carry out further development. The developed prototype provides a subset of the total functionality. The prototype makes it possible to demonstrate and assess new techniques for the investigation of robotic assembly problems such as the integration of robot programming and sequence planning. This prototype can also be refined into an industrial prototype.

## 7.3 Future Research

The integration of robotic assembly programming and sequence planning is an area of research that is still in a research phase. There are still many problems in developing practical off-line robot-based automation systems. Following are some directions for future research.

- **Improving off-line programming models** The control model should have exactly the same behavior of a real controller. It is necessary to use the algorithms that drive real controllers. Solutions to problems such as the singularity of a robot are controller-specific. The system has to have different robot simulators and to allow a simulation of different controllers within a workcell.
- **Incorporating other planning approaches** The presented approach assumes that one of the assembly task sequences is known in advance. In less structured, more dynamic manufacturing systems, the context of an assembly plan is not known in advance and there is a need to conduct planning before the teaching process. In such cases, the presented approach needs to incorporate 'editor' or 'full-automatic' approaches for pre-planning.
- **Integration of planning** The integration of assembly and task planning is receiving great attentions recently[58]. *Task planning* determines a sequence of tasks (the course of actions) to be performed by a robot to achieve some desirable state in the robot's world (the goal). Task planners often deal with high level issues and thus rely on lower level planners to complete lower level tasks. In particular, gross motion planning for assembly operations and fine motion planning in general are the two technologies that are usually utilized in assembly and task planning systems.
- **Complexity study** In the example given in chapter 6, the number of distinct feasible assembly plans can be very large even for assemblies made up of a small number of parts. If the planning task involves many objects, the complexity

will be very high. A proper selection of all possible operations is therefore needed. Moreover, the time needed to compute a possible plan to realize a task increases rapidly when the complexity of the task increases. If the planning problem is not restricted by other means, the complexity of planning can be a NP-complete problem. When it happens, there is no guarantee that a plan will be generated within a given time. Since a complete enumeration of assembly plan is prohibitive in most real applications, finding systematic ways to narrow down the alternatives is crucial for the automatic planning of assembly [54]. To overcome the problems associated with the complexity of planning for exception handling, the following requirements must be met: (1) The planning space must be limited. This implies that a limited number of sequences is needed to bring the robot system from its initial state into the desired goal state. (2) Heuristic knowledge concerning the application environment is needed to guide the planning activity. In this way, the planner can be tailored to the application and thus yield a better performance.

- **High level robot operations** Testing a simulator with operations other than ‘pick-and-place’ is much more complicated. This thesis does not work on general algorithms to deduct all the feasible sequences for other types of robotic operations. Existing technologies still face great difficulties when attempting to create an integrated model for general-purpose operations. Although many industrial automation applications do not require the sophistication of robot operations, the introduction of different robots’ operations — ranging from ‘side-insert’ to ‘multi-hands’ operations — points to a strong need of a wide

application domain.

- **Finding optimal sequences** In this thesis, optimal planning issues are not considered. In order to explore the ‘automatic re-programming’ feature, a planner capable of finding the optimal assembly sequence within a set of feasible assembly sequences is needed [54]. Two approaches are being tried. One, qualitative approach, is to develop rules to eliminate assembly plans that include difficult tasks or awkward intermediate sub-assemblies. Another quantitative approach is to introduce an evaluation function to compute the merit of assembly plans. It may be based on the cost of the resources needed to complete an assembly, on the total time required, or on the difficulty of execution. A combination of the two approaches may attain the advantages of both.

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## Testing Results of The Pendulum Assembly Task

## A.2 All Feasible Sequences

SeqNo: 1	Table	Splatel	Spacer1	Spacer2	Spacer3	Spacer4	Shaft	Spice	Lever	Splate2
SeqNo: 2	Table	Splatel	Spacer1	Spacer2	Spacer3	Spacer4	Shaft	Lever	Spice	Splate2
SeqNo: 3	Table	Splatel	Spacer1	Spacer2	Spacer3	Spacer4	Spice	Shaft	Lever	Splate2
SeqNo: 4	Table	Splatel	Spacer1	Spacer2	Spacer3	Shaft	Spacer4	Spice	Lever	Splate2
SeqNo: 5	Table	Splatel	Spacer1	Spacer2	Spacer3	Shaft	Spacer4	Lever	Spice	Splate2
SeqNo: 6	Table	Splatel	Spacer1	Spacer2	Spacer3	Shaft	Spice	Spacer4	Lever	Splate2
SeqNo: 7	Table	Splatel	Spacer1	Spacer2	Spacer3	Shaft	Spice	Lever	Spacer4	Splate2
SeqNo: 8	Table	Splatel	Spacer1	Spacer2	Spacer3	Shaft	Lever	Spacer4	Spice	Splate2
SeqNo: 9	Table	Splatel	Spacer1	Spacer2	Spacer3	Shaft	Lever	Spice	Spacer4	Splate2
SeqNo: 10	Table	Splatel	Spacer1	Spacer2	Spacer3	Spice	Spacer4	Shaft	Lever	Splate2
SeqNo: 11	Table	Splatel	Spacer1	Spacer2	Spacer3	Spice	Shaft	Spacer4	Lever	Splate2
SeqNo: 12	Table	Splatel	Spacer1	Spacer2	Spacer3	Spice	Shaft	Lever	Spacer4	Splate2
SeqNo: 13	Table	Splatel	Spacer1	Spacer2	Spacer4	Spacer3	Shaft	Spice	Lever	Splate2
SeqNo: 14	Table	Splatel	Spacer1	Spacer2	Spacer4	Spacer3	Shaft	Lever	Spice	Splate2
SeqNo: 15	Table	Splatel	Spacer1	Spacer2	Spacer4	Spacer3	Spice	Shaft	Lever	Splate2
SeqNo: 16	Table	Splatel	Spacer1	Spacer2	Spacer4	Shaft	Spacer3	Spice	Lever	Splate2
SeqNo: 17	Table	Splatel	Spacer1	Spacer2	Spacer4	Shaft	Spacer3	Lever	Spice	Splate2
SeqNo: 18	Table	Splatel	Spacer1	Spacer2	Spacer4	Shaft	Spice	Spacer3	Lever	Splate2
SeqNo: 19	Table	Splatel	Spacer1	Spacer2	Spacer4	Shaft	Spice	Lever	Spacer3	Splate2
SeqNo: 20	Table	Splatel	Spacer1	Spacer2	Spacer4	Shaft	Lever	Spacer3	Spice	Splate2

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SeqNo : 538	Table	=	Splate1	=	Spacer4	=	Spacer2	=	Shaft	=	Lever	=	Spacer1	=	Spacer3	=	Spiece	=	Splate2
SeqNo : 539	Table	=	Splate1	=	Spacer4	=	Spacer2	=	Shaft	=	Lever	=	Spacer1	=	Spiece	=	Spacer3	=	Splate2
SeqNo : 540	Table	=	Splate1	=	Spacer4	=	Spacer2	=	Shaft	=	Lever	=	Spacer3	=	Spacer1	=	Spiece	=	Splate2
SeqNo : 541	Table	=	Splate1	=	Spacer4	=	Spacer3	=	Spacer1	=	Spacer2	=	Shaft	=	Spiece	=	Lever	=	Splate2
SeqNo : 542	Table	=	Splate1	=	Spacer4	=	Spacer3	=	Spacer1	=	Spacer2	=	Shaft	=	Lever	=	Spiece	=	Splate2
SeqNo : 543	Table	=	Splate1	=	Spacer4	=	Spacer3	=	Spacer1	=	Spacer2	=	Spiece	=	Shaft	=	Lever	=	Splate2
SeqNo : 544	Table	=	Splate1	=	Spacer4	=	Spacer3	=	Spacer1	=	Shaft	=	Spacer2	=	Lever	=	Spiece	=	Splate2
SeqNo : 545	Table	=	Splate1	=	Spacer4	=	Spacer3	=	Spacer1	=	Shaft	=	Spacer2	=	Spiece	=	Lever	=	Splate2
SeqNo : 546	Table	=	Splate1	=	Spacer4	=	Spacer3	=	Spacer1	=	Shaft	=	Lever	=	Spacer2	=	Spiece	=	Splate2
SeqNo : 547	Table	=	Splate1	=	Spacer4	=	Spacer3	=	Spacer2	=	Spacer1	=	Shaft	=	Spiece	=	Lever	=	Splate2
SeqNo : 548	Table	=	Splate1	=	Spacer4	=	Spacer3	=	Spacer2	=	Spacer1	=	Shaft	=	Lever	=	Spiece	=	Splate2
SeqNo : 549	Table	=	Splate1	=	Spacer4	=	Spacer3	=	Spacer2	=	Spacer1	=	Spiece	=	Shaft	=	Lever	=	Splate2
SeqNo : 550	Table	=	Splate1	=	Spacer4	=	Spacer3	=	Spacer2	=	Shaft	=	Spacer1	=	Lever	=	Spiece	=	Splate2
SeqNo : 551	Table	=	Splate1	=	Spacer4	=	Spacer3	=	Spacer2	=	Shaft	=	Spacer1	=	Spiece	=	Lever	=	Splate2
SeqNo : 552	Table	=	Splate1	=	Spacer4	=	Spacer3	=	Spacer2	=	Shaft	=	Lever	=	Spacer1	=	Spiece	=	Splate2
SeqNo : 553	Table	=	Splate1	=	Spacer4	=	Spacer3	=	Shaft	=	Spacer1	=	Spacer2	=	Lever	=	Spiece	=	Splate2
SeqNo : 554	Table	=	Splate1	=	Spacer4	=	Spacer3	=	Shaft	=	Spacer1	=	Spacer2	=	Spiece	=	Lever	=	Splate2
SeqNo : 555	Table	=	Splate1	=	Spacer4	=	Spacer3	=	Shaft	=	Spacer1	=	Lever	=	Spacer2	=	Spiece	=	Splate2
SeqNo : 556	Table	=	Splate1	=	Spacer4	=	Spacer3	=	Shaft	=	Spacer2	=	Spacer1	=	Lever	=	Spiece	=	Splate2
SeqNo : 557	Table	=	Splate1	=	Spacer4	=	Spacer3	=	Shaft	=	Spacer2	=	Spacer1	=	Spiece	=	Lever	=	Splate2
SeqNo : 558	Table	=	Splate1	=	Spacer4	=	Spacer3	=	Shaft	=	Spacer2	=	Lever	=	Spacer1	=	Spiece	=	Splate2
SeqNo : 559	Table	=	Splate1	=	Spacer4	=	Spacer3	=	Shaft	=	Lever	=	Spacer1	=	Spacer2	=	Spiece	=	Splate2
SeqNo : 560	Table	=	Splate1	=	Spacer4	=	Spacer3	=	Shaft	=	Lever	=	Spacer2	=	Spacer1	=	Spiece	=	Splate2
SeqNo : 561	Table	=	Splate1	=	Spacer4	=	Shaft	=	Spacer1	=	Spacer2	=	Spacer3	=	Lever	=	Spiece	=	Splate2
SeqNo : 562	Table	=	Splate1	=	Spacer4	=	Shaft	=	Spacer1	=	Spacer2	=	Spacer3	=	Spiece	=	Lever	=	Splate2
SeqNo : 563	Table	=	Splate1	=	Spacer4	=	Shaft	=	Spacer1	=	Spacer2	=	Lever	=	Spacer3	=	Spiece	=	Splate2
SeqNo : 564	Table	=	Splate1	=	Spacer4	=	Shaft	=	Spacer1	=	Spacer2	=	Lever	=	Spiece	=	Spacer3	=	Splate2
SeqNo : 565	Table	=	Splate1	=	Spacer4	=	Shaft	=	Spacer1	=	Spacer2	=	Spiece	=	Spacer3	=	Lever	=	Splate2
SeqNo : 566	Table	=	Splate1	=	Spacer4	=	Shaft	=	Spacer1	=	Spacer2	=	Spiece	=	Lever	=	Spacer3	=	Splate2
SeqNo : 567	Table	=	Splate1	=	Spacer4	=	Shaft	=	Spacer1	=	Spacer3	=	Spacer2	=	Lever	=	Spiece	=	Splate2
SeqNo : 568	Table	=	Splate1	=	Spacer4	=	Shaft	=	Spacer1	=	Spacer3	=	Spacer2	=	Spiece	=	Lever	=	Splate2
SeqNo : 569	Table	=	Splate1	=	Spacer4	=	Shaft												

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SeqNo : 820 Table = Splat1 Shaft = Lever = Spacer3 = Spacer3 = Spacer1 = Splice = Spacer4 = Splat2
SeqNo : 821 Table = Splat1 Shaft = Lever = Spacer2 = Spacer3 = Spacer4 = Splat1 = Splice = Splat2
SeqNo : 822 Table = Splat1 Shaft = Lever = Spacer2 = Spacer4 = Spacer1 = Spacer3 = Splice = Splat2
SeqNo : 823 Table = Splat1 Shaft = Lever = Spacer2 = Spacer4 = Spacer1 = Splice = Spacer3 = Splat2
SeqNo : 824 Table = Splat1 Shaft = Lever = Spacer2 = Spacer4 = Spacer3 = Spacer1 = Splice = Splat2
SeqNo : 825 Table = Splat1 Shaft = Lever = Spacer3 = Spacer1 = Spacer2 = Spacer4 = Splice = Splat2
SeqNo : 826 Table = Splat1 Shaft = Lever = Spacer3 = Spacer1 = Spacer2 = Splice = Spacer4 = Splat2
SeqNo : 827 Table = Splat1 Shaft = Lever = Spacer3 = Spacer1 = Spacer4 = Spacer2 = Splice = Splat2
SeqNo : 828 Table = Splat1 Shaft = Lever = Spacer3 = Spacer3 = Spacer1 = Spacer4 = Splice = Splat2
SeqNo : 829 Table = Splat1 Shaft = Lever = Spacer3 = Spacer2 = Spacer1 = Splice = Spacer4 = Splat2
SeqNo : 830 Table = Splat1 Shaft = Lever = Spacer3 = Spacer2 = Spacer4 = Spacer1 = Splice = Splat2
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SeqNo : 832 Table = Splat1 Shaft = Lever = Spacer3 = Spacer4 = Spacer2 = Spacer1 = Splice = Splat2
SeqNo : 833 Table = Splat1 Shaft = Lever = Spacer4 = Spacer1 = Spacer2 = Spacer3 = Splice = Splat1
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SeqNo : 835 Table = Splat1 Shaft = Lever = Spacer4 = Spacer1 = Spacer3 = Spacer2 = Splice = Splat2
SeqNo : 836 Table = Splat1 Shaft = Lever = Spacer4 = Spacer2 = Spacer1 = Spacer3 = Splice = Splat2
SeqNo : 837 Table = Splat1 Shaft = Lever = Spacer4 = Spacer2 = Spacer1 = Splice = Spacer3 = Splat2
SeqNo : 838 Table = Splat1 Shaft = Lever = Spacer4 = Spacer2 = Spacer3 = Spacer1 = Splice = Splat2
SeqNo : 839 Table = Splat1 Shaft = Lever = Spacer4 = Spacer3 = Spacer1 = Spacer2 = Splice = Splat2
SeqNo : 840 Table = Splat1 Shaft = Lever = Spacer5 = Spacer3 = Spacer2 = Spacer1 = Splice = Splat2
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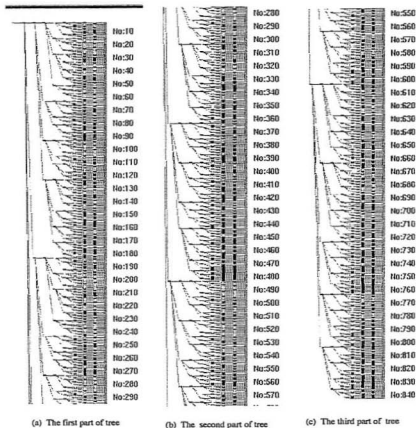


Figure A.1: A feasible assembly tree for the pendulum assembly task.







